A Math P1
(4047)

i) $y = 2x^2 - 6x - 4$ — (1)

$y + 2x = 12$ — (2)

Subst (1) into (2):

$2x^2 - 6x - 4 + 2x = 12$

$2x^2 - 4x - 16 = 0$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = 4$ or $x = -2$

When $x = 4$, $y = 12 - 2(4)$
$= 4$

When $x = -2$, $y = 12 - 2(-2)$
$= 16$

Coordinates of points of intersection
$= (4, 4)$ & $(-2, 16)$

ii) $y = 2x^2 - kx - 4$ — (3)

$y + 2x = 12$ — (4)

Subst: (3) into (4):

$2x^2 - kx - 4 + 2x = 12$

$2x^2 + (2-k)x - 16 = 0$

$D = (2-k)^2 - 4(2)(-16)$
$= 4 - 4k + k^2 + 128$
$= k^2 - 4k + 132$
$= k^2 - 4k + 2^2 + 132 - 2^2$
$= (k-2)^2 + 128$

Since $(k-2)^2 \geq 0$ for any value of k,

$\therefore (k-2)^2 + 128 > 0$

$\therefore$ line intersects curve at 2 distinct points. (shown)

2) $16+4\sqrt{3} = (\sqrt{6}+\sqrt{2})^2 \times h$

$h = \dfrac{16+4\sqrt{3}}{6+2\sqrt{12}+2}$

$= \dfrac{16+4\sqrt{3}}{8+2\sqrt{12}}$

$= \dfrac{16+4\sqrt{3}}{8+4\sqrt{3}}$

$= \dfrac{(16+4\sqrt{3})(8-4\sqrt{3})}{64-16(3)}$

$= \dfrac{128-64\sqrt{3}+32\sqrt{3}-48}{16}$

$= \dfrac{80-32\sqrt{3}}{16}$

$= 5-2\sqrt{3}$ *

3a i) Between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

ii) Between 0 and π.

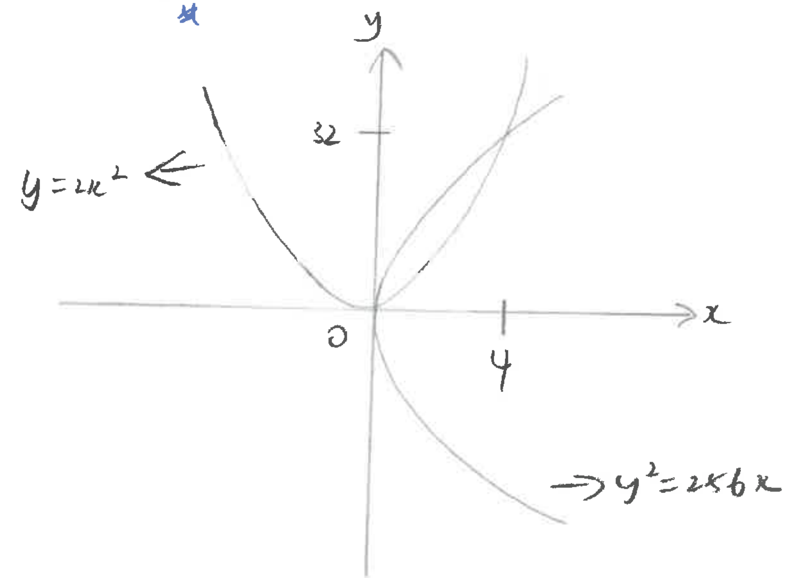
b) $a=-1$ *

$\frac{1}{b} = \frac{2\pi}{12\pi} = \frac{1}{6}$

$b=6$ *

$c = 1-(-1) = 2$ *

4i)



$(2x^2)^2 = 256x$

$4x^4 = 256x$

$x(x^3-64)=0$

$x=0$ or $x^3=64$

$x=4$

When $x=4$, $y=32$

4ii) $(0,0)$ & $(4,32)$

$$\text{gradient} = \frac{32-0}{4-0}$$

$$= 8$$

$$y = 8x + c$$

At $(0,0)$: $0 = c$

$$\therefore y = 8x$$ A

5)

$$\begin{array}{r|l} & \quad 2 \\ \hline x^2+x-6 & 2x^2+4x-31 \\ & -)\ 2x^2+2x-12 \\ \hline & \quad 2x-19 \end{array}$$

$$\therefore \frac{2x^2+4x-31}{x^2+x-6} = 2 + \frac{2x-19}{(x+3)(x-2)}$$

$$\frac{2x-19}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$2x-19 = A(x-2) + B(x+3)$$

Let $x=2$: $2(2)-19 = 5B$

$$B = -3$$

Let $x=-3$: $2(-3)-19 = -5A$

$$A = 5$$

$$\frac{2x^2+4x-31}{x^2+x-6} = 2 + \frac{5}{x+3} - \frac{3}{x-2}$$ A

6i) $\frac{p+6}{2} = 4$

$p+6=8$

$p = 2$ #

ii) $y = ax^2+bx+c$

At $(2,0)$: $0 = 4a+2b+c$ — ①

At $(6,0)$: $0 = 36a+6b+c$ — ②

At $(4,-6)$: $-6 = 16a+4b+c$ — ③

Substi ① into ②: $4a+2b = 36a+6b$

$32a = -4b$

$b = -8a$ — ④

Substi ④ into ③: $-6 = 16a+4(-8a)+c$

$= -16a+c$ — ⑤

Substi ④ into ①: $0 = 4a+2(-8a)+c$

$c = 12a$ — ⑥

Substi ⑥ into ⑤: $-6 = -16a+12a$

$4a = 6$

$a = 1.5$ #

When $a = 1.5$, $b = -8(1.5)$

$= -12$ #

$c = 12(1.5)$

$= 18$ #

iii) $0 < q < 6$ #

7i) Taking downwards & leftwards to be positive.

Cyclist 1: $v = 5$

$s = 5t + C$

$s = 5t$ (since $s = 0$ at $t = 0$)

$OP = 5t$

Cyclist 2: $v = -10$

$s = -10t + c$

$s = -10t + 100$ (since $s = 100$ at $t = 0$)

$OQ = 100 - 10t$

$\therefore s = \sqrt{(5t)^2 + (100 - 10t)^2}$

$= \sqrt{25t^2 + 100t^2 - 2000t + 10000}$

$= \sqrt{125t^2 - 2000t + 10000}$

$= \sqrt{125(t^2 - 16t + 80)}$ (shown)

ii) $s = (\sqrt{125})(t^2 - 16t + 80)^{\frac{1}{2}}$

$\frac{ds}{dt} = (\sqrt{125})(\frac{1}{2})(t^2 - 16t + 80)^{-\frac{1}{2}}(2t - 16)$

$= \frac{(\sqrt{125})(t - 8)}{\sqrt{t^2 - 16t + 80}}$

iii) $\frac{ds}{dt} = 0$

$\therefore \frac{\sqrt{125}(t - 8)}{\sqrt{t^2 - 16t + 80}} = 0$

$t = 8$

$\frac{ds}{dt} = \sqrt{125}(t - 8)(t^2 - 16t + 80)^{-\frac{1}{2}}$

$\frac{d^2s}{dt^2} = \sqrt{125}(t - 8)(-\frac{1}{2})(t^2 - 16t + 80)^{-\frac{3}{2}}(2t - 16)$

$+ \sqrt{125}(t^2 - 16t + 80)^{-\frac{1}{2}}(1)$

$= \sqrt{125}(t^2 - 16t + 80)^{-\frac{3}{2}}\left[-(t - 8)^2 + (t^2 - 16t + 80)\right]$

$= \frac{(\sqrt{125})\left[-(t^2 - 16t + 64) + (t^2 - 16t + 80)\right]}{(t^2 - 16t + 80)^{\frac{3}{2}}}$

$= \frac{16\sqrt{125}}{(t^2 - 16t + 80)^{\frac{3}{2}}} > 0$ when $t = 8$

$\therefore s$ is min. at $t = 8$.

$s = \sqrt{2000} = 44.7\text{m}$ (3 s.f.)

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8i) Let coordinates of B be (a, b).

$2y + 3x = 45$

$y = \frac{45 - 3x}{2}$

gradient of $BC = -1.5$

gradient of $AB = \frac{2}{3}$

When $y = 0$, $3x = 45$

$x = 15$

$\therefore$ Coordinates of $C = (15, 0)$

$-1.5 = \frac{b - 0}{a - 15}$

$b = -1.5(a - 15)$ — ①

$\frac{2}{3} = \frac{b - 6}{a + 2}$

$2a + 4 = 3b - 18$

$2a = 3b - 22$ — ②

Subst. ① into ②: $2a = 3(-1.5a + 22.5) - 22$

$= -4.5a + 67.5 - 22$

$= -4.5a + 45.5$

$a = 7$

$\therefore b = -1.5(7 - 15)$

$= 12$

Coordinates of $B = (7, 12)$

ii) Coordinates of $M = \left(\frac{-2 + 15}{2}, \frac{6 + 0}{2}\right)$

$= (6.5, 3)$

Let coordinates of D be (c, d).

$\vec{AB} = \vec{DC}$

$\begin{pmatrix} 7 \\ 12 \end{pmatrix} - \begin{pmatrix} -2 \\ 6 \end{pmatrix} = \begin{pmatrix} 15 \\ 0 \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix}$

$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 15 \\ 0 \end{pmatrix} - \begin{pmatrix} 7 \\ 12 \end{pmatrix} + \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -6 \end{pmatrix}$

$\therefore$ Coordinates of $D = (6, -6)$

9i) $y = 2 - x^2 - \frac{16}{x^2}$

$\frac{dy}{dx} = -2x - 16(-2)(x^{-3})$

$= -2x + \frac{32}{x^3}$

$\therefore -2x + \frac{32}{x^3} = 0$

$-2x^4 + 32 = 0$

$2x^4 = 32$

$x^4 = 16$

$x = 2$ or $x = -2$

When $x=2$, $y = 2 - 4 - \frac{16}{4}$

$= -6$

When $x=-2$, $y = 2 - 4 - \frac{16}{4}$

$= -6$

$\therefore$ Coordinates $= (2,-6)$ & $(-2,-6)$

ii) $\frac{d^2y}{dx^2} = -2 + 32(-3)(x^{-4})$

$= -2 - \frac{96}{x^4}$

When $x=2$, $\frac{d^2y}{dx^2} = -8 < 0$ (max.)

When $x=-2$, $\frac{d^2y}{dx^2} = -8 < 0$ (max.)

Both points are maximum.

10 i) $\frac{d}{dx}\left(\frac{\ln x}{x^2}\right) = \frac{x^2\left(\frac{1}{x}\right) - (\ln x)(2x)}{x^4}$

$= \frac{x - 2x\ln x}{x^4}$

$= \frac{1}{x^3} - \frac{2\ln x}{x^3}$ (shown)

ii) $\int \frac{\ln x}{x^3}\,dx = \frac{1}{2}\int \frac{2\ln x}{x^3}\,dx$

$= \frac{1}{2}\int \frac{1}{x^3}\,dx - \frac{1}{2}\left(\frac{\ln x}{x^2}\right) + c$

$= \frac{1}{2}\left(\frac{1}{-2x^2}\right) - \frac{1}{2}\left(\frac{\ln x}{x^2}\right) + c$

$= -\frac{1}{4x^2} - \frac{\ln x}{2x^2} + c$

iii) $f'(x) = \frac{\ln x}{x^3}$

$f(x) = -\frac{1}{4x^2} - \frac{\ln x}{2x^2} + c$

At $(1, \frac{3}{4})$: $\frac{3}{4} = -\frac{1}{4} - \frac{\ln 1}{2} + c$

$c = 1$

$\therefore f(x) = -\frac{1}{4x^2} - \frac{\ln x}{2x^2} + 1$

11a) RHS: $\frac{1-\sin\theta}{1+\sin\theta} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$

$= \frac{1 - 2\sin\theta + \sin^2\theta}{\cos^2\theta}$

$= \sec^2\theta - \frac{2\sin\theta}{\cos^2\theta} + \tan^2\theta$

$= \sec^2\theta - 2\tan\theta\sec\theta + \tan^2\theta$

$= (\sec\theta - \tan\theta)^2$

$=$ LHS (proven)

b i) $k = \frac{2\pi}{60}$

$= \frac{\pi}{30}$ rad/min

ii) $d > 40$

$|80 \sin kt| > 40$

$80 \sin kt > 40$ or $80 \sin kt < -40$

$\sin kt > \frac{1}{2}$ | $\sin kt < -\frac{1}{2}$

$\sin kt = \frac{1}{2}$ | $\sin kt = -\frac{1}{2}$

$kt = \frac{\pi}{6}, \frac{5}{6}\pi$ | $kt = \frac{7}{6}\pi, \frac{11}{6}\pi$

$t = 5, 25$ | $t = 35, 55$

$5 < t < 25$ | $35 < t < 55$

$\therefore d > 40$ for a total of 40 min in each hour.

12 i) $\int_0^{\frac{\pi}{6}} f(x)\,dx = \left[\sin x + k\cos 2x\right]_0^{\frac{\pi}{6}}$

$= \left(\frac{1}{2} + k\left(\frac{1}{2}\right)\right) - (k)$

$= \frac{1}{2} + \frac{1}{2}k - k$

$= \frac{1}{2} - \frac{1}{2}k$

$\therefore \frac{1}{2} - \frac{1}{2}k = \frac{3}{4}$

$\frac{1}{2}k = -\frac{1}{4}$

$k = -\frac{1}{2}$ (shown)

ii) $\int f(x)\,dx = \sin x - \frac{1}{2}\cos 2x + c$

$f(x) = \frac{d}{dx}\left(\sin x - \frac{1}{2}\cos 2x + c\right)$

$= \cos x + \sin 2x$

iii) $y = \cos x + \sin 2x$

$\frac{dy}{dx} = -\sin x + 2\cos 2x$

gradient (when $x = \frac{\pi}{6}$) $= -\sin\frac{\pi}{6} + 2\cos\frac{\pi}{3}$

$= -\frac{1}{2} + 1$

$= \frac{1}{2}$

gradient of normal $= -2$

$\therefore y = -2x + c$

when $x = \frac{\pi}{6}$, $y = \cos\frac{\pi}{6} + \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

$= \sqrt{3}$

$\therefore$ At $\left(\frac{\pi}{6}, \sqrt{3}\right)$: $\sqrt{3} = -2\left(\frac{\pi}{6}\right) + c$

$c = \sqrt{3} + \frac{\pi}{3}$

$\therefore y = -2x + \sqrt{3} + \frac{\pi}{3}$