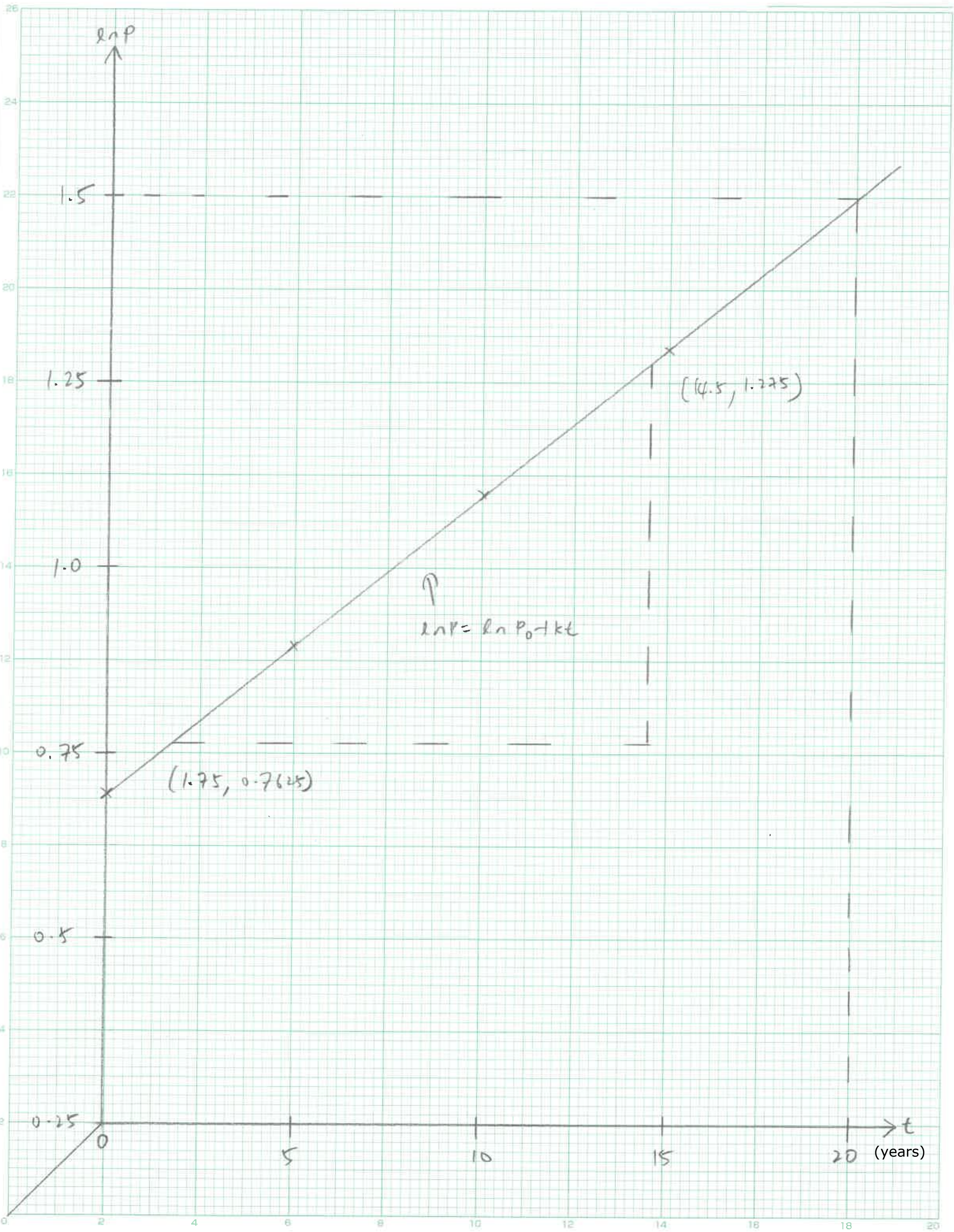
(1)


ln P
1.5
1.25
1.0
0.75
0.5
0.25
0
5
10
15
20
t
(years)
(14.5, 1.275)
(1.75, 0.7625)
ln P = ln P0 + kt

i) $P = P_0 e^{kt}$

$\ln P = \ln P_0 + \ln e^{kt}$

$\ln P = \ln P_0 + kt$, where k = gradient

(Plot $\ln P$ against t.) $\ln P_0$ = vertical intercept

t (years)	0	5	10	15
$\ln P$	0.693	0.892	1.10	1.29

ii) $\ln P_0 = 0.693$

$P_0 = \$2.00$

$k = \dfrac{1.275 - 0.7825}{14.5 - 1.75}$

$= 0.0402$ (3 s.f.)

iii) From the graph, (extrapolation)

when $t = 20$ (year 2015), $\ln P = 1.5$

$P = 4.48$ (3 s.f.)

2i) $(1-2x)^2(1-px)^6 = (1-4x+4x^2)[1^6+6(1^5)(-px)+15(1^4)(-px)^2+\dots]$

$=(1-4x+4x^2)(1-6px+15p^2x^2+\dots)$

$=\dots+4x^2+24px^2+15p^2x^2+\dots$

$\therefore\ 4+24p+15p^2=16$

$15p^2+24p-12=0$

$5p^2+8p-4=0$

$(5p-2)(p+2)=0$

$p=\frac{2}{5}$ or $p=-2$

ii) When $p=\frac{2}{5}$, $T_4=\binom{6}{3}(1)^3(-\frac{2}{5}x)^3$

$=20(-\frac{8}{125}x^3)$

$=-\frac{32}{25}x^3$

Coeff. of $x^3=-\frac{32}{25}$

$=-1\frac{7}{25}$

When $p=-2$, $T_4=\binom{6}{3}(1)^3(2x)^3$

$=20(8x^3)$

$=160x^3$

Coeff. of $x^3=160$

3i) $\cos 3x=\cos(2x+x)$

$=\cos 2x\cos x-\sin 2x\sin x$

$=(1-2\sin^2 x)(\cos x)-(2\sin x\cos x)(\sin x)$

$=\cos x-2\cos x\sin^2 x-2\sin^2 x\cos x$

$=\cos x-4\sin^2 x\cos x$

$=\cos x(1-4\sin^2 x)$ (shown)

ii) $2\cos 3x=15\sin x\cos x$

$2\cos x(1-4\sin^2 x)-15\sin x\cos x=0$

$\cos x(2-8\sin^2 x-15\sin x)=0$

$\cos x=0$ or $2-8\sin^2 x-15\sin x=0$

$x=90°, 270°$

$8\sin^2 x+15\sin x-2=0$

$(8\sin x-1)(\sin x+2)=0$

$\sin x=\frac{1}{8}$ or $\sin x=-2$ (N.A.)

$\alpha=7.181°$

$x=7.2°, 172.8°$

4i) $\alpha+\beta=-2$

$\alpha\beta=5$

$\alpha^3+\beta^3=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$

$=(-2)[(-2)^2-3(5)]$

$=22$ *

ii) $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$

$=\frac{22}{5^2}$

$=\frac{22}{25}$

$\frac{\alpha}{\beta^2}\times\frac{\beta}{\alpha^2}=\frac{1}{\alpha\beta}$

$=\frac{1}{5}$

Equation: $x^2-\frac{22}{25}x+\frac{1}{5}=0$

$25x^2-22x+5=0$ *

5i) Let $\angle PBA=x^\circ$.

$\angle ACB=x^\circ$ ($\angle$s in alt. segment)

$\angle DBC=x^\circ$ (isos. $\triangle BDC$)

$\angle CDB=180^\circ-2x$

$\angle ADB=180^\circ-(180^\circ-2x)$ ($\angle$s on a str. line)

$=2x$

$\angle PAB=x^\circ$ (isos. $\triangle PAB$; tangent from an ext. pt.)

$\angle APB=180^\circ-2x$

$\therefore \angle APB+\angle ADB=180^\circ-2x+2x$

$=180^\circ$ * (shown)

ii) $\angle BDP=\angle PAB=x^\circ$ ($\angle$s in same seg.)

Since $\angle BDP=\angle DBC=x^\circ$,

this means that 'alt. $\angle$s' holds and $PD\,//\,BC$. * (proven)

6i) $\frac{dy}{dx} = (x-2)(3)(2x-5)^2(2) + (2x-5)^3(1)$

$= 6(x-2)(2x-5)^2 + (2x-5)^3$

$= (2x-5)^2[6(x-2) + 2x-5]$

$= (6x-12+2x-5)(2x-5)^2$

$= (8x-17)(2x-5)^2$ *

ii) $\frac{dy}{dx} < 0$

$\therefore (8x-17)(2x-5)^2 < 0$

Since $(2x-5)^2 \geq 0$ for all values of x,

$8x-17 < 0$ for $\frac{dy}{dx} < 0$ to hold.

$x < 2\frac{1}{8}$ *

iii) $\frac{dy}{dt} = 0.35$ units/s (when $x=3$)

When $x=3$, $\frac{dy}{dx} = [8(3)-17][2(3)-5]^2$

$= 7$

$\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$

$= \frac{1}{7} \times 0.35$

$= 0.05$

x is increasing at the rate of 0.05 units per seconds. *

iv) $z = y^2$

$\frac{dz}{dy} = 2y$

$= 2(x-2)(2x-5)^3$

$= 2(3-2)(6-5)^3$, when $x=3$

$= 2$

$\frac{dz}{dt} = \frac{dz}{dy} \times \frac{dy}{dt}$

$= 2 \times 0.35$

$= 0.7$

$\frac{dz}{dt} = k\left(\frac{dy}{dt}\right)$

$0.7 = k(0.35)$

$k = 2$

$\therefore \frac{dz}{dt} = 2\frac{dy}{dt}$ * when $x=3$. (shown)

7i) $2^{2x-1} = 2^{x+2} - 6$

$(2^x)^2 \cdot \frac{1}{2} = 2^x \cdot 4 - 6$

$\therefore \frac{u^2}{2} = 4u - 6$

$u^2 - 8u + 12 = 0$ ✱

ii) $(u-6)(u-2) = 0$

$u = 6$ or $u = 2$

$\therefore 2^x = 6$ $\quad 2^x = 2$

$x = \frac{\lg 6}{\lg 2}$ $\quad x = 1$ ✱

$= 2.6$ ✱
(1d.p.)

iii) Let $u = 2^x$.

$\therefore 2^{2x-1} = 2^{x+2} - k$

$\frac{u^2}{2} - 4u = -k$

$u^2 - 8u + 2k = 0$

$D = (-8)^2 - 4(1)(2k)$
$= 64 - 8k$

When $D < 0$, $64 - 8k < 0$
$k > 8$ ✱

8i) $f(3) = 3^3 - 3(3)^2 + 4(3) - 12$
$= 0$

$\therefore (x-3)$ is a factor of $f(x)$

$$
\begin{array}{r l}
 & x^2 + 4 \\
x-3 \,\big) & x^3 - 3x^2 + 4x - 12 \\
-) & x^3 - 3x^2 \\
\hline
 & 0 + 4x - 12 \\
-) & 4x - 12 \\
\hline
 & 0
\end{array}
$$

$\therefore f(x) = (x-3)(x^2+4)$ ✱

ii) $f(x) = 0$

$\therefore (x-3)(x^2+4) = 0$

$x = 3$ or $x^2 = -4$ (rej.)

Since x^2 cannot be negative, $f(x) = 0$ only has 1 real root (3). ✱

8iii) $y = x^3 - 3x^2 + 4x - 12 + kx$

$\frac{dy}{dx} = 3x^2 - 6x + 4 + k$

$\frac{d^2y}{dx^2} = 6x - 6$

$\therefore 6x - 6 = 0$

$x = 1$

when $x = 1$, $\frac{dy}{dx} = 3(1)^2 - 6(1) + 4 + k$

$= 0$

$\therefore 3 - 6 + 4 + k = 0$

$k = -1$

9i) $\frac{dy}{dx} = 3x^2 + 4x - 3$

Gradient at $A = 3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) - 3$

$= 1$

ii) Gradient at $B = 1$

$\therefore 3x^2 + 4x - 3 = 1$

$3x^2 + 4x - 4 = 0$

$(3x - 2)(x + 2) = 0$

$x = \frac{2}{3}$ or $x = -2$

$\therefore$ x-coordinate of $B = -2$

iii) Total shaded area $= \int_{-2}^{0} (x^3 + 2x^2 - 3x)\,dx + \left| \int_{0}^{\frac{2}{3}} (x^3 + 2x^2 - 3x)\,dx \right|$

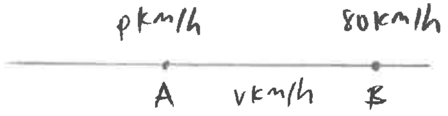
$= \left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{3x^2}{2} \right]_{-2}^{0} + \left| \left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{3x^2}{2} \right]_{0}^{\frac{2}{3}} \right|$

$= 0 - \left(\frac{16}{4} - \frac{16}{3} - \frac{12}{2} \right) + \left| \frac{4}{81} + \frac{16}{81} - \frac{2}{3} - 0 \right|$

$= 7\frac{1}{3} + \left| -\frac{34}{81} \right|$

$= 7\frac{61}{81}$ units^2

10 i)



At A, $t=0$

$\therefore p = 30e^0 + 20$

$= 50$

ii) At B, $v = 80$

$\therefore 80 = 30e^{25t} + 20$

$e^{25t} = 2$

$25t = \ln 2$

$t = \frac{\ln 2}{25}$

$= 0.02773\ h$

$= 100s$ (nearest s)

iii) dist. between A & B $= \int_0^{\frac{\ln 2}{25}} (30e^{25t} + 20)\ dt$

$= \left[\frac{30e^{25t}}{25} + 20t\right]_0^{\frac{\ln 2}{25}}$

$= \left(\frac{60}{25} + \frac{20\ln 2}{25}\right) - \left(\frac{30}{25} + 0\right)$

$= 1\frac{1}{5} + \frac{4\ln 2}{5}$

$= \left(\frac{6+4\ln 2}{5}\right)$ km

iv) $a = \frac{dv}{dt}$

$= 30(25)e^{25t}$

$= 750e^{25t}$

11i) $x^2+y^2-4x-2y=95$

$x^2-4x+2^2+y^2-2y+1^2=95+2^2+1^2$

$(x-2)^2+(y-1)^2=100$

Coordinates of $A=(2,1)$

Radius $=\sqrt{100}$

$=10$ units

ii) When $x=10$, $10^2+y^2-4(10)-2y=95$

$y^2-2y-35=0$

$(y-7)(y+5)=0$

$y=7$ or $y=-5$

$\therefore$ Point $P(10,7)$ lies on C_1. (Shown)

iii) Gradient of $AP=\frac{7-1}{10-2}$

$=\frac{3}{4}$

Gradient of tangent at $P=-\frac{4}{3}$

Equation: $y=-\frac{4}{3}x+c$

At point P, $7=-\frac{4}{3}(10)+c$

$c=20\frac{1}{3}$

$\therefore y=-\frac{4}{3}x+20\frac{1}{3}$

iv) $AP=10$ units ; radius $=5$ units

Let centre of C_2 be O.

Coordinates of $O=\left(\frac{2+10}{2},\frac{1+7}{2}\right)$

$=(6,4)$

Equation of C_2: $(x-6)^2+(y-4)^2=25$

v) Equation of tangent to C_2 at P:

$y=-\frac{4}{3}x+20\frac{1}{3}$

(7)