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9 (i) Gradient of AC $= \frac{0-10}{5-0}$

$= -2$

Gradient of OB $= -\frac{1}{-2}$

$= \frac{1}{2}$

(since the two lines in a kite are perpendicular; this is a property of kites)

$y = \frac{1}{2}x + c$

$y = \frac{1}{2}x$ (since y-int is 0)

(ii) OC = OB = 10 units

OA = AB = 5 units

~~Midpt of OB~~

Equation of AC:

$y = -2x + c$

$= -2x + 10$ (int. is 10)

Midpt of OB is the intersection ~~is~~ of OB and AC.

$\frac{1}{2}x = -2x + 10$

$\frac{5}{2}x = 10$

$x = 4$

$y = \frac{1}{2}(4)$

$= 2$

Therefore, coordinates of B is $(4\times2,\ 2\times2) = (8, 4)$

10 (i) $3x + 2\pi r = 20$

$2\pi r = 20 - 3x$

$r = \frac{20-3x}{2\pi}$

(ii) $A = \frac{1}{2}x^2 \sin 60^\circ + \pi r^2$

$= \frac{1}{2}x^2 \frac{\sqrt{3}}{2} + \pi\left(\frac{20-3x}{2\pi}\right)^2$

$= \frac{\sqrt{3}x^2}{4} + \frac{\pi(20-3x)^2}{4\pi^2}$

$= \frac{\sqrt{3}\pi x^2}{4\pi} + \frac{(20-3x)^2}{4\pi}$

$= \frac{\sqrt{3}\pi x^2 + (20-3x)^2}{4\pi}$

(iii) $\frac{dA}{dx} = \frac{1}{4\pi}\left[2\sqrt{3}\pi x + (2)(20-3x)(-3)\right]$

$= \frac{1}{4\pi}\left[2\sqrt{3}\pi x - 120 + 18x\right]$

At the stationary value, $\frac{dA}{dx} = 0$

$\frac{1}{4\pi}\left[2\sqrt{3}\pi x - 120 + 18x\right] = 0$

$2\sqrt{3}\pi x + 18x = 120$

$x(2\sqrt{3}\pi + 18) = 120$

$x = \frac{120}{2\sqrt{3}\pi + 18}$

$= 4.15472239$

≈ 4.15 m

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10 (iv) $\frac{d^2A}{dx^2} = \frac{1}{4\pi}\left(2\sqrt{3}\pi + 18\right)$

which is obviously > 0

i.e. this stationary value is a minimum.

The gardener may be disappointed because making two good flower beds give a ~~total~~ small total area A, whereas, maximising total area will require one of the beds to be sacrificed.

11 (i) $f'(x) = \frac{10x-9}{2x-3}$

$= \frac{10x-15+6}{2x-3}$

$= \frac{5(2x-3)}{2x-3} + \frac{6}{2x-3}$

$= 5 + \frac{6}{2x-3}$

(ii) For $x > \frac{3}{2}$, $2x-3 > 0$

$\frac{6}{2x-3} > 0$

$5 + \frac{6}{2x-3} > 0$

Therefore, $f(x)$ is an increasing function (since its gradient function $f'(x)$ is positive).

(iii) $f''(x) = \frac{(2x-3)\cdot\frac{d}{dx}(10x-9) - (10x-9)\cdot\frac{d}{dx}(2x-3)}{(2x-3)^2}$

$= \frac{(2x-3)(10) - (10x-9)(2)}{(2x-3)^2}$

$= \frac{20x-30-20x+18}{(2x-3)^2}$

$= -\frac{12}{(2x-3)^2}$

$(2x-3)^2 \geq 0$ but we cannot equate denominators to 0

i.e. $(2x-3)^2 > 0$

$\frac{12}{(2x-3)^2} > 0$

$-\frac{12}{(2x-3)^2} < 0$

$f''(x) < 0$

Therefore, $f'(x)$ is a decreasing function.

(iv) $f(x) = \int f'(x)\,dx$

$= \int \frac{10x-9}{2x-3}\,dx$

$= \int \left(5 + \frac{6}{2x-3}\right)dx$

$= 5x + \frac{6\ln(2x-3)}{2} + c$ where c is an arbitrary constant

$= 5x + 3\ln(2x-3) + c$

Since $f(2) = 8$,

$8 = 5(2) + 3\ln(2\cdot 2 - 3) + c$

$c = -2 - 3\ln 1 = -2$

$\therefore f(x) = 5x + 3\ln(2x-3) - 2$