Subject/Topic: A Maths P2 2018 Date:

10(i) Gradient of NP $= \frac{2\sqrt{7-3k}-0}{k-(-5)} = \frac{2\sqrt{7-3k}}{k+5}$

Gradient of PT $= -\frac{1}{\text{grad of NP}} = -\frac{k+5}{2\sqrt{7-3k}}$

Now, for the curve $y = 2\sqrt{7-3x}$,

$\frac{dy}{dx} = 2\left(\frac{1}{2}\right)(7-3x)^{-\frac{1}{2}} \cdot \frac{d}{dx}(7-3x)$

$= \frac{1}{\sqrt{7-3x}}(-3)$

$= -\frac{3}{\sqrt{7-3x}}$

At $x = k$, $\frac{dy}{dx} = -\frac{3}{\sqrt{7-3k}}$

Since $\frac{dy}{dx}$ at $x = k$ gives the gradient of the curve at $x = k$,

$-\frac{3}{\sqrt{7-3k}} = -\frac{k+5}{2\sqrt{7-3k}}$

$-3 = -\frac{k+5}{2}$

$6 = k+5$

$k = 1$

(ii) Since the gradient of PT $= -\frac{3}{\sqrt{7-3k}}$,

$y = -\frac{3}{\sqrt{7-3(1)}}x + c$

$= -\frac{3}{2}x + c$

At $(1, 2\sqrt{7-3(1)})$ i.e. $(1, 4)$,

$4 = -\frac{3}{2}(1) + c$

$c = \frac{11}{2}$

$y = -\frac{3}{2}x + \frac{11}{2}$

At T, $y = 0$

$-\frac{3}{2}x + \frac{11}{2} = 0$

$\frac{3}{2}x = \frac{11}{2}$

$x = \frac{11}{3}$

Therefore, the x-coordinate of T is $\frac{11}{3}$.

(iii) Area of shaded region

$= \int_1^{11/3}\left(-\frac{3}{2}x + \frac{11}{2}\right)dx - \int_1^{7/3} \ldots$

Where the curve meets the x-axis,

$2\sqrt{7-3x} = 0$

$\sqrt{7-3x} = 0$

$7-3x = 0$

$x = \frac{7}{3}$

Area of shaded region

$= \int_1^{11/3}\left(-\frac{3}{2}x + \frac{11}{2}\right)dx - \int_1^{7/3} 2\sqrt{7-3x}\,dx$

$= \left[-\frac{3}{4}x^2 + \frac{11}{2}x\right]_1^{11/3} - \left[\frac{2(7-3x)^{3/2}}{3/2 \cdot -3}\right]_1^{7/3}$

$= \left[-\frac{3}{4}x^2 + \frac{11}{2}x\right]_1^{11/3} + \frac{4}{9}\left[(7-3x)^{3/2}\right]_1^{7/3}$

$= \left[-\frac{3}{4}\left(\frac{11}{3}\right)^2 + \frac{11}{2}\left(\frac{11}{3}\right)\right] - \left[-\frac{3}{4}(1)^2 + \frac{11}{2}(1)\right]$

$+ \frac{4}{9}\left[(7-3(\frac{7}{3}))^{3/2} - (7-3(1))^{3/2}\right]$

$= \left[-\frac{121}{12} + \frac{121}{6}\right] - \left[-\frac{3}{4} + \frac{11}{2}\right]$

$+ \frac{4}{9}\left[0 - 4^{3/2}\right]$

$= \frac{121}{12} - \frac{19}{4} + \frac{4}{9}(-8)$

$= \frac{16}{9}$ units^2

11(i) Gradient of AB × Gradient of BC

$= \frac{8-4}{9-1} \times \frac{12-8}{7-9}$

$= \frac{4}{8} \times \frac{4}{-2}$

$= -1$

Therefore, $A\hat{B}C = 90°$

(ii) Because the angle opposite the diameter in a triangle inside a semicircle is 90°.

(iii) Centre of circle $= \left(\frac{1+7}{2}, \frac{4+12}{2}\right)$

$= (4, 8)$

Radius of circle $= \frac{1}{2}\sqrt{(7-1)^2 + (12-4)^2}$

$= \frac{1}{2} \cdot 10$

$= 5$ units

The circle has equation

$(x-4)^2 + (y-8)^2 = 5^2$

$(x-4)^2 + (y-8)^2 = 25$

(iv) B lies directly above the centre of the circle. Of course B is the maximum point of the circle, hence the tangent at B is parallel to the y-axis.

(v) Since tangent ⊥ diameter,

gradient of tangent at C $= -\frac{1}{\frac{12-8}{7-1}} = -\frac{3}{4}$

$y = -\frac{3}{4}x + c$

$12 = -\frac{3}{4}(7) + c \Rightarrow c = \frac{69}{4}$

$y = -\frac{3}{4}x + \frac{69}{4} \Rightarrow 4y = -3x + 69$

Tuition classes for English, Math (including E Maths & A Maths), Science (including combined science [phy/chem/bio]), Physics, Chemistry, Biology, Social Studies/Geography/ History and Principles of Accounts (POA). Secondary 1 to Secondary 4.