Subject/Topic: A Maths P2 2018 Date:

5(iii) $DE = 1200\cos\theta - 800\sin\theta$

$= \sqrt{1200^2 + 800^2}\cos\left(\theta + \tan^{-1}\frac{800}{1200}\right)$

$= \sqrt{2080000}\cos\left(\theta + \tan^{-1}\frac{2}{3}\right)$

$= 1442.22051\cos(\theta + 33.69006753^\circ)$

$\approx 1440\cos(\theta + 33.7^\circ)$

Since $DE = 200$ m,

$\sqrt{2080000}\cos\left(\theta + \tan^{-1}\frac{2}{3}\right) = 200$

$\cos\left(\theta + \tan^{-1}\frac{2}{3}\right) = \frac{200}{\sqrt{2080000}}$

$\theta + \tan^{-1}\frac{2}{3} = \cos^{-1}\frac{200}{\sqrt{2080000}}$ (reject 4th quad)

$\theta = \cos^{-1}\frac{200}{\sqrt{2080000}} - \tan^{-1}\frac{2}{3}$

$= 48.33874817^\circ$

$\approx 48.3^\circ$

6(i) $\frac{d}{dx}(x\cos x) = x\frac{d}{dx}\cos x + \cos x\frac{d}{dx}x$

$= x(-\sin x) + \cos x(1)$

$= \cos x - x\sin x$

(ii) $\int x\sin x\,dx = -\int -x\sin x\,dx$

$= -\left[\int(\cos x - x\sin x - \cos x)\,dx\right]$

$= -\left[\int(\cos x - x\sin x)\,dx - \int\cos x\,dx\right]$

$= -\int(\cos x - x\sin x)\,dx + \int\cos x\,dx$

$= -x\cos x + \sin x + C_1$

$= \sin x - x\cos x + C_1$

where C_1 is an arbitrary constant

(iii) $\frac{d}{dx}(x^2\sin x) = x^2\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x^2)$

$= x^2\cos x + 2x\sin x$

$\int x^2\cos x\,dx = \int[x^2\cos x + 2x\sin x - 2x\sin x]\,dx$

$= \int(x^2\cos x + 2x\sin x)\,dx - 2\int x\sin x\,dx$

$= x^2\sin x - 2(\sin x - x\cos x + C_1) + C_2$

$= x^2\sin x - 2\sin x + 2x\cos x + C_1 + C_2$

$= x^2\sin x - 2\sin x + 2x\cos x + C_3$

where C_2 and $C_3 = C_1 + C_2$ are arbitrary constants

7(i) Velocity $= \frac{d}{dt}\left[840(1 - e^{-t/80}) - 2t\right]$

$= 840\left[-e^{-t/80}\cdot\frac{d}{dt}\left(-\frac{t}{80}\right)\right] - 2$

$= 840\left(-e^{-t/80}\right)\left(-\frac{1}{80}\right) - 2$

$= \frac{840}{80}e^{-t/80} - 2$

$= \frac{21}{2}e^{-t/80} - 2$ m/s

Acceleration $= \frac{d}{dt}\left(\frac{21}{2}e^{-t/80} - 2\right)$

$= \frac{21}{2}e^{-t/80}\left(-\frac{1}{80}\right)$

$= -\frac{21}{160}e^{-t/80}$ m/s^2

When $t = 10$, $v = \frac{21}{2}e^{-10/80} - 2$

$= 7.266217477$

≈ 7.27 m/s

When $t = 10$, $a = -\frac{21}{160}e^{-10/80}$

$= -0.1158277185$

≈ -0.116 m/s^2

(ii) The girl is slowing down as she cycles up the hill.