Subject/Topic: A Maths P2 2018 Date:

1(i) $x^2 + 3x + 5 = 0$

Since the roots are $\alpha$ and $\beta$,

$\alpha + \beta = -\frac{3}{1}$

$= -3$

$\alpha\beta = \frac{5}{1}$

$= 5$

$(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

$= 5 + (-3) + 1$

$= 3$

(ii) For the equation whose roots are $\frac{2}{\alpha+1}$ and $\frac{2}{\beta+1}$,

Sum of roots $= \frac{2}{\alpha+1} + \frac{2}{\beta+1}$

$= \frac{2(\beta+1) + 2(\alpha+1)}{(\alpha+1)(\beta+1)}$

$= \frac{2(\alpha+\beta+2)}{(\alpha+1)(\beta+1)}$

$= \frac{2(-3+2)}{3}$

$= \frac{2(-1)}{3}$

$= -\frac{2}{3}$

Product of roots $= \left(\frac{2}{\alpha+1}\right)\left(\frac{2}{\beta+1}\right)$

$= \frac{4}{(\alpha+1)(\beta+1)}$

$= \frac{4}{3}$

The quadratic equation is given by

$x^2 - \left(-\frac{2}{3}x\right) + \frac{4}{3} = 0$

$x^2 + \frac{2}{3}x + \frac{4}{3} = 0$

$3x^2 + 2x + 4 = 0$

2 $(2+ax)^6 = 2^6 + \binom{6}{1}2^{6-1}(ax)^1 + \binom{6}{2}2^{6-2}(ax)^2 + \ldots$

$= 64 + 192ax + 240a^2x^2 + \ldots$

(Terms beyond $x^2$ are not required here)

$(1-4x)(2+ax)^6 = (1-4x)(64 + 192ax + 240a^2x^2 + \ldots)$

$= 64 + 192ax + 240a^2x^2 - 256x - 768ax^2 + \ldots$

$= 64 + (192a - 256)x + (240a^2 - 768a)x^2 + \ldots$

Since the first three terms in the expansion are $64 - 160x + bx^2$,

$64 + (192a - 256)x + (240a^2 - 768a)x^2 = 64 - 160x + bx^2$

Equating the coefficients of $x$, $192a - 256 = -160$

$192a = 96$

$a = \frac{1}{2}$

Equating the coefficients of $x^2$, $240a^2 - 768a = b$

$b = 240\left(\frac{1}{2}\right)^2 - 768\left(\frac{1}{2}\right)$

$= -324$

3 Since $AP = AQ$, $A\hat{P}Q = A\hat{Q}P$.

Now, $A\hat{P}B + B\hat{P}Q = A\hat{P}Q$

$= A\hat{Q}P$

$= B\hat{C}P + Q\hat{P}C$ (since ext. $\angle$ of $\triangle$ = sum of int. opp. $\angle$s)

However, $A\hat{P}B = B\hat{C}P$ (alt. seg. thm)

So, $A\hat{P}B + B\hat{P}Q = A\hat{P}B + Q\hat{P}C$

$B\hat{P}Q = Q\hat{P}C$

and therefore, PQ bisects $B\hat{P}C$.