



$$1) \frac{dy}{dx} = \int (8-6x) dx \\ = 8x - \frac{6x^2}{2} + C$$

$$= 8x - 3x^2 + C$$

$$\text{When } x=2, \frac{dy}{dx} = 2.$$

$$\therefore 8(2) - 3(2)^2 + C = 2$$

$$C = -1$$

$$\frac{dy}{dx} = 8x - 3x^2 - 1$$

$$y = \int (8x - 3x^2 - 1) dx \\ = \frac{8x^2}{2} - \frac{3x^3}{3} - x + d \\ = 4x^2 - x^3 - x + d$$

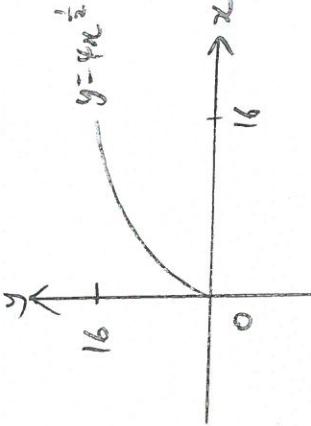
$$\text{At } (2, 8): 4(2)^2 - 2^3 - 2 + d = 8$$

$$d = 2$$

Eqn of curve: $y = 4x^2 - x^3 - x + 2$

$$2) \text{ When } x=16, y = 4(16)^{\frac{1}{2}}$$

$$= 16$$



$$\text{i)} \quad y = 4x^{\frac{1}{2}} - 1$$

$$4y = 4x + 4$$

$$y = \frac{1}{4}x + 1 \quad \text{---(2)}$$

$$\text{Sub (2) into (1): } \frac{7}{4}x + 1 = 4x^{\frac{1}{2}}$$

$$7x + 4 = 16x^{\frac{1}{2}}$$

$$(x^{\frac{1}{2}} - 2)(7x^{\frac{1}{2}} - 2) = 0$$

$$x^{\frac{1}{2}} = 2$$

$$x = 4$$

$$y = 8 \quad y = 1\frac{1}{7}$$

(1)

$$3) \quad Y = mx + c$$

$$\frac{1}{y} = m\left(\frac{1}{Jx}\right) + c$$

when $x = 0.04$ & $y = 0.25$,

$$\frac{1}{0.25} = m\left(\frac{1}{J0.04}\right) + c$$

$$4 = 5m + c$$

$$c = 4 - 5m \quad \text{--- (1)}$$

when $x = 1.00$ & $y = 0.50$,

$$\frac{1}{0.50} = m\left(\frac{1}{J1.00}\right) + c$$

$$2 = m + c \quad \text{--- (2)}$$

Solve (1) into (2) : $2 = m + 4 - 5m$

$$4m = 2$$

$$m = \frac{1}{2}$$

$$\therefore c = 4 - 5\left(\frac{1}{2}\right)$$

$$= 1\frac{1}{2}$$

$$\text{when } x = 9, \quad \frac{1}{y} = \frac{1}{2}\left(\frac{1}{Jx}\right) + 1\frac{1}{2} \quad \text{--- (2)} \\ = 1\frac{2}{3}$$

$$4) \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{7} \quad \parallel \quad \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{7}$$

$$\frac{\alpha + \beta}{\alpha \beta} = \frac{3}{7} \quad \parallel \quad \frac{1}{\alpha \beta} = \frac{1}{7}$$

$$\begin{aligned} \therefore \alpha \beta &= 7 \\ \alpha + \beta &= 3 \end{aligned}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\begin{aligned} &= (3)^2 - 2(7) \\ &= -5 \end{aligned}$$

$$\begin{aligned} \alpha^2 \beta^2 &= (\alpha \beta)^2 \\ &= 7^2 \\ &= 49 \end{aligned}$$

$$\text{Eqn: } x^2 + 5x + 49 = 0$$

$$\begin{aligned}
 5i) LHS &= \frac{\sec x + \cos x}{\sec x - \cos x} \\
 &= \frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\frac{1}{\cos x} - \frac{1}{\sin x}} \\
 &= \frac{\sin x + \cos x}{\cos x \sin x} \\
 &= \frac{\sin x + \cos x}{\cos x \sin x} \\
 &= \frac{\cos x (\tan x + 1)}{\cos x (\tan x - 1)} \\
 &= \frac{\tan x + 1}{\tan x - 1} \\
 &= RHS \quad (\text{shown})
 \end{aligned}$$

$$5ii) \quad \frac{\tan x + 1}{\tan x - 1} = \frac{5}{2}$$

$$2\tan x + 2 = 5\tan x - 5$$

$$3\tan x = 7$$

$$\tan x = \frac{7}{3} \quad [Q1, Q3]$$

$$x = 1.1659 \quad (5s.f.)$$

$$x = 1.17, 4.31 \quad (2s.f.)$$

(3)

$$6i) \quad 288 = 6x + 4y$$

$$4y = 288 - 6x$$

$$y = 72 - 1.5x \quad \text{--- (1)}$$

$$A = y \times 3x$$

$$= (72 - 1.5x) \times 3x$$

$$= 216x - \frac{9}{2}x^2 \quad (\text{shown})$$

$$ii) \quad \frac{dA}{dx} = 0$$

$$\therefore 216 - 9x = 0$$

$$9x = 216$$

$$x = 24$$

$$y = 72 - 1.5(24)$$

$$= 36$$

The dimensions are 24m & 36m respectively.

$$7i) \quad A = \frac{1}{2}(AB)(AC) \sin \angle BAC$$

$$\frac{1}{4}(9 + \sqrt{3}) = \frac{1}{2}(\sqrt{3} + 1)(AC) \sin 60^\circ$$

$$\frac{9}{4} + \frac{\sqrt{3}}{4} = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)(AC)$$

$$= \left(\frac{3}{4} + \frac{\sqrt{3}}{4}\right)(AC)$$

$$9 + \sqrt{3} = (3 + \sqrt{3})(AC)$$

$$AC = \frac{9 + \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{(9 + \sqrt{3})(3 - \sqrt{3})}{9 - 3}$$

$$= \frac{27 - 9\sqrt{3} + 3\sqrt{3} - 3}{6}$$

$$= \frac{24 - 6\sqrt{3}}{6}$$

$$= (4 - \sqrt{3})\text{ cm}$$

(4)

7(i) Using cosine rule,

$$\begin{aligned} BC^2 &= AB^2 + AC^2 - 2(AB)(AC) \cos \angle BAC \\ &= (\sqrt{3}+1)^2 + (4-\sqrt{3})^2 - 2(\sqrt{3}+1)(4-\sqrt{3}) \cos 60^\circ \\ &= 3 + 2\sqrt{3} + 1 + 16 - 8\sqrt{3} + 3 - (4\sqrt{3} - 3 + 4 - \sqrt{3}) \\ &= 2\sqrt{3} - 6\sqrt{3} - 3\sqrt{3} - 1 \end{aligned}$$

$$= (22 - 9\sqrt{3}) \text{ cm}^2$$

$$8(ii) \quad \frac{6+11x-5x^2}{3x^3-x^2+2x-9} = \frac{6+11x-5x^2}{(3x-1)(x^2+9)}$$

$$\begin{aligned} &= \frac{A}{3x-1} + \frac{Bx+C}{x^2+9} \\ &= \frac{A(x^2+9) + (Bx+C)(3x-1)}{3x^3-x^2+2x-9} \end{aligned}$$

$$\therefore 6+11x-5x^2 = A(x^2+9) + (Bx+C)(3x-1)$$

$$\text{Let } x = \frac{1}{3} : 9 \frac{1}{9} = 9 \frac{1}{9} A$$

$$\begin{aligned} A &= 1 \\ \text{Let } x = 0 : 6 &= 9A + C(-1) \\ 6 &= 9 - C \\ C &= 3 \end{aligned}$$

$$\begin{aligned} \text{Let } x = 1 : 12 &= 10A + (B+C)(2) \\ 12 &= 10 + 2B + 6 \end{aligned}$$

$$\therefore \frac{3x^3-x^2+2x-9}{3x-1} = x^2+9$$

$$\begin{aligned} 2B &= -4 \\ B &= -2 \end{aligned}$$

$$\therefore \frac{6+11x-5x^2}{3x^3-x^2+2x-9} = \frac{1}{3x-1} + \frac{3-2x}{x^2+9}$$

(S)

(6)



PENCILTUTOR SCHOOL (PTE) LTD.
Co. Reg. No. 200601708E
Blk 102, #02-135
Yishun Avenue 5
Singapore 760102

PENCILTUTOR SCHOOL (PTE) LTD.
Co. Reg. No. 200601708E
Blk 102, #02-135
Yishun Avenue 5
Singapore 760102



$$\text{i)} \quad v = \int 10 dt \\ = 10t + C$$

When $t=0, v=0 : C=0$

$$\therefore v = 10t$$

At X, $t=4$.

$$\therefore v = 40 \text{ m/s}$$

$$\text{ii)} \quad s = \int (10t) dt$$

$$= \frac{10t^2}{2} + d$$

$$= 5t^2 + d$$

When $t=0, s=0 : d=0$

$$\therefore s = 5t^2$$

$$\begin{aligned} OX &= 5(4)^2 \\ &= 80 \text{ m} \end{aligned}$$

$$\text{iii)} \quad v = \int (10 - kT) dT$$

$$= 10T - \frac{kT^2}{2} + q$$

When $T=0, v=40 : 40=q$

$$\therefore v = 10T - \frac{kT^2}{2} + 40$$

When $T=3, v = 10(3) - \frac{k(3)^2}{2} + 40$

$$= 70 - \frac{9}{2}k \\ = 0$$

$$\therefore 70 - \frac{9}{2}k = 0$$

$$\frac{9}{2}k = 70$$

$$k = \frac{140}{9} \quad (\text{shown})$$

$$10) i) \angle PBA = \angle ACB \quad (\text{As in alt. segment})$$

$$\angle DAC = \angle ACB \quad (\text{alt. } \cancel{\text{As}}; AD \parallel BC)$$

$$\therefore \angle PBA = \angle DAC \quad \text{(shown)}$$

$$ii) \text{ Let } \angle PBA = x^\circ.$$

$$\therefore \angle DAC = x^\circ.$$

$$\angle ACD = 90^\circ \quad (\cancel{\text{As in semicircle}})$$

$$\angle ACD = 180^\circ - 90^\circ - x^\circ$$

$$\angle ABC = 180^\circ - (90^\circ - x^\circ)$$

$$= 90^\circ + x^\circ \quad (\text{sum of } \cancel{\text{As in } \triangle})$$

$$\angle CBT = 180^\circ - x^\circ - (90^\circ + x^\circ)$$

$$= 90^\circ - 2x^\circ \quad (\cancel{\text{As on a str. line}})$$

$$= 90^\circ - 2x \times (\cancel{\angle PBA}) \quad \text{(shown)}$$

$$ii) \frac{dy}{dx} = \frac{(x-1)(2) - (2x+1)(1)}{(x-1)^2}$$

$$= \frac{2x-2 - 2x-1}{(x-1)^2}$$

$$= \frac{-3}{(x-1)^2}$$

Since $(x-1)^2 \geq 0$ and the numerator is a non-zero, $\frac{dy}{dx}$ can never be zero.
Hence, curve has no turning points.

$$i) \frac{x-1 \sqrt{2x+1}}{(-1)^2 - 2x - 2} = 2t \frac{3}{x-1}$$

... continue on next page.

7

(11.i) ... continue from previous page.

Area of shaded region

$$= \text{Area of trapezium} - \int_2^4 \left(2 + \frac{3}{x-1} \right) dx$$

$$= \frac{1}{2} (3+5)(2) - \left[2x + 3 \ln(x-1) \right]_2^4$$

$$= 8 - \left[(8 + 3 \ln 3) - (4 + 3 \ln 1) \right]$$

$$= 8 - [8 + 3 \ln 3 - 4]$$

$$= 8 - 4 - 3 \ln 3$$

$$= 4 - 3 \ln 3$$

$$= 0.704 \text{ units}^2 \quad (\text{3.s.f.})$$

$$(12.i) \quad x^2 + y^2 + 8x - 24y + 96 = 0$$

$$x^2 + 8x + y^2 - 24y = -96$$

$$(x+4)^2 + (y-12)^2 = -96 + 4^2 + 12^2$$

$$(x+4)^2 + (y-12)^2 = 64 \quad \text{--- (3)}$$

$$\text{Centre} = (-4, 12)$$

$$\text{Radius} = 8 \text{ units}$$

$$3y + 4x = k \quad \text{--- (1)}$$

$$\text{Sub } (-4, 12) \text{ into eq (1):}$$

$$3(-12) + 4(-4) = k$$

$$k = 20 \quad \text{A}$$

(8)

$$12 \text{ ii) } y = -\frac{4}{3}x + \frac{20}{3} \quad \text{---} \textcircled{2}$$

When $y=0 : 4x = 20$

$$x = 5$$

$$S(5, 0)$$

$$\text{Sub } \textcircled{2} \text{ into } \textcircled{3} : (x+4)^2 + \left(-\frac{4}{3}x + \frac{20}{3} - 12\right)^2 = 64$$

$$-(x+4)^2 + \left(-\frac{4}{3}x - 5\frac{1}{3}\right)^2 = 64$$

$$x^2 + 8x + 16 + 1\frac{7}{9}x^2 + 14\frac{2}{9}x + 28\frac{4}{9} = 64$$

$$2\frac{7}{9}x^2 + 22\frac{2}{9}x - 19\frac{5}{9} = 0$$

$$25x^2 + 200x - 176 = 0$$

$$(5x - 4)(5x + 44) = 0$$

$$x = \frac{4}{5} \quad \text{or} \quad x = -8\frac{4}{5}$$

(req. as $-4 < x < 5$)

$$\text{When } x = \frac{4}{5}, y = -\frac{4}{3}\left(\frac{4}{5}\right) + \frac{20}{3}$$

$$= 5\frac{3}{5}$$

$$R\left(\frac{4}{5}, 5\frac{3}{5}\right)$$



$$12 \text{ ii) } \text{Length of RS} = \sqrt{(5 - \frac{4}{5})^2 + (0 - 5\frac{3}{5})^2}$$

$$= \sqrt{49}$$

= 7 units

(9)

PENCILTUTOR SCHOOL (PTE) LTD.
Co. Reg. No. 200601708E
Blk 102, #02-135
Yishun Avenue 5
Singapore 760102

PENCILTUTOR SCHOOL (PTE) LTD.
Co. Reg. No. 200601708E
Blk 102, #02-135
Yishun Avenue 5
Singapore 760102



PENCILTUTOR[®]