

$$1) \quad y = e^{-x} x^2$$

$$\frac{dy}{dx} = e^{-x} (2x) + x^2 (-e^{-x})$$

$$= e^{-x} (2x - x^2)$$

$$\frac{d^2y}{dx^2} = e^{-x} (2-2x) + (2x-x^2)(-e^{-x})$$

$$= e^{-x} (2-2x-2x+x^2)$$

$$= e^{-x} (2-4x+x^2)$$

$$\therefore k = e^x [e^{-x} (2-4x+x^2) + 2e^{-x} (2x-x^2) + e^{-x} x^2]$$

$$= 2-4x+x^2 + 4x-2x^2+x^2$$

$$= 2$$

$$2i) \quad \frac{d}{dx} (\tan x - x) = \sec^2 x - 1$$

$$= \tan^2 x \quad (\text{shown})$$

$$ii) \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec^2 x + 5 \tan^2 x) dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec^2 x + 5 \sec^2 x - 5) dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (6 \sec^2 x - 5) dx$$

$$= \left[6 \tan x - 5x \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \left[6 \tan \frac{\pi}{3} - 5 \left(\frac{\pi}{3} \right) \right] - \left[6 \tan \frac{\pi}{6} - 5 \left(\frac{\pi}{6} \right) \right]$$

$$= 6 \left(\frac{3}{\sqrt{3}} \right) - \frac{5\pi}{3} - 6 \left(\frac{1}{\sqrt{3}} \right) + \frac{5\pi}{6}$$

$$= \frac{18}{\sqrt{3}} - \frac{6}{\sqrt{3}} - \frac{10\pi}{6} + \frac{5\pi}{6}$$

$$= \frac{12}{\sqrt{3}} - \frac{5\pi}{6}$$

$$= \frac{12\sqrt{3}}{3} - \frac{5\pi}{6}$$

$$= 4\sqrt{3} - \frac{5\pi}{6}$$

$$\therefore a = 4 \quad \& \quad b = -\frac{5}{6}$$

(1)

$$\begin{aligned}
 3(i) \quad T_{4r} &= \binom{9}{r} (px^3)^{9-r} \left(\frac{1}{x}\right)^r \\
 &= \binom{9}{r} (p^{9-r})(x^3)^{9-r} (x^{-1})^r \\
 &= \binom{9}{r} (p^{9-r}) (x^{27-3r-r}) \\
 &= \binom{9}{r} (p^{9-r}) (x^{27-4r})
 \end{aligned}$$

power of $x = 27 - 4r$

Since $4r$ is divisible by 2, it is always even.

Subtracting an even number from an odd one (27) will always yield an odd number.

$$3(ii) \quad 27 - 4r = 7$$

$$r = 4$$

$$\begin{aligned}
 T_5 &= \binom{9}{4} (p^5) (x^5) \\
 &= 126 p^5 x^5
 \end{aligned}$$

$$r = 5$$

$$\begin{aligned}
 T_6 &= \binom{9}{5} (p^4) (x^7) \\
 &= 126 p^4 x^7
 \end{aligned}$$

$$\therefore 126 p^5 = 2 (126 p^4)$$

$$p^5 = 2 p^4$$

$$p = 2 \quad a$$

(2)



$$\text{iv) } \frac{dy}{dx} = 0$$

$$\therefore 6(-\frac{1}{2})x^{-\frac{3}{2}} + 1 = 0$$

$$-3x^{-\frac{3}{2}} = -1$$

$$x^{-\frac{3}{2}} = \frac{1}{3}$$

$$x^{\frac{3}{2}} = 3$$

$x^3 = 9$ (shown)

$$\text{ii) } \frac{dy}{dx} \text{ at } x=1 : 6(-\frac{1}{2})(1)^{-\frac{3}{2}} + 1 = -2$$

$$\frac{dy}{dx} \text{ at } x=4 : 6(-\frac{1}{2})(4)^{-\frac{3}{2}} + 1 = \frac{5}{8}$$

\therefore Tangent at A : $y - 7 = -2(x - 1)$

$$y = -2x + 9 \quad \text{--- ①}$$

Tangent at B : $y - 7 = \frac{5}{8}(x - 4)$

$$y = \frac{5}{8}x + 4\frac{1}{2} \quad \text{--- ②}$$

(Continue on next Pg...)

$$\text{Sub ① into ② : } -2x + 9 = \frac{5}{8}x + 4\frac{1}{2}$$

$$-16x + 72 = 5x + 36$$

$$21x = 36$$

$$x = 1\frac{5}{7} < \sqrt[3]{9} \approx 2.08$$

\therefore x-coordinate of P is less than $\sqrt[3]{9}$
the x-coordinate of M.

(3)

$$\text{i)} \log_5 \left(\frac{5x+1}{x+1} \right) = \log_5 5 + \log_5 \frac{1}{7}$$

$$= \log_5 \left(\frac{5}{7} \right)$$

$$\therefore \frac{5x+1}{x+1} = \frac{5}{7}$$

$$7x - 7 = 5x + 5$$

$$2x = 12$$

$$x = 6$$

$$\text{6i) } \frac{dy}{dx} = m$$

$$\therefore 18x + 2m + 1 = m$$

$$x = \frac{-m-1}{18} \quad \text{--- (1)}$$

Sub eqn of line into eqn of curve:

$$mx + c = 9x^2 + (2m+1)x + 1 + c$$

$$mx = 9x^2 + 2mx + x + 1$$

$$9x^2 + (m+1)x + 1 = 0 \quad \text{--- (2)}$$

$$\begin{aligned} \text{sub (1) into (2): } & 9\left(\frac{-m-1}{18}\right)^2 + (m+1)\left(\frac{-m-1}{18}\right) + 1 = 0 \\ & 9\left(\frac{m^2+2m+1}{324}\right) + \left(\frac{-m^2-2m-1}{18}\right) + 1 = 0 \\ & m^2 + 2m + 1 - 2m^2 - 4m - 2 + 36 = 0 \\ & m^2 + 2m - 35 = 0 \\ & (m-5)(m+7) = 0 \\ m = 5 & \text{ or } m = -7 \\ & \text{at } (2 \text{ s.f.)} \end{aligned}$$

(4)

(6ii) $m = 5$

$$\text{At } (-2, 19): 19 = 9(-2)^2 + (2 \times 5+1)(-2) + 1 + c$$

$$c = 4$$

$$\text{Eq (1): } x = \frac{-5-1}{18}$$

$$= -\frac{1}{3}$$

$$\text{When } x = -\frac{1}{3}, y = 9\left(-\frac{1}{3}\right)^2 + (10+1)\left(-\frac{1}{3}\right) + 1 + 4$$

$$= 1 - \frac{11}{3}$$

$$= 2\frac{1}{3}$$

$$\therefore P\left(-\frac{1}{3}, 2\frac{1}{3}\right)$$

$$\text{7(i) } \frac{1}{2}p = 100e^{-k(5730)}$$

Since $p = 100$ when $t = 0$,

$$\frac{1}{2}(100) = 100e^{-k(5730)}$$

$$e^{-5730k} = \frac{1}{2}$$

$$-5730k = \lambda n \frac{1}{2}$$

$$k = \frac{\lambda n \frac{1}{2}}{-5730}$$

$$= 1.2097 \times 10^{-4} \quad (\text{5s.f.})$$

$$= 1.21 \times 10^{-4} \quad (\text{3s.f.})$$

$$\text{ii) when } t = 8000, p = 100e^{-8000(1.2097 \times 10^{-4})}$$

$$= 38.0 \quad (\text{3s.f.})$$

(5)

(6iii) L is a vertical line which intersects the curve at the minimum point. \star
(i.e. line of symmetry) \star

7b) When $s = 2.4$, $2.4 = \lg \frac{I}{c}$

$$= \lg I - \lg c$$

$$\lg c = \lg I - 2.4$$

$$\text{When } I_{\text{new}} = 50I, \quad S_{\text{new}} = \lg \frac{50I}{c}$$

$$= \lg 50 + \lg I - \lg c$$

$$= \lg 50 + \lg I - (\lg I + 2.4)$$

$$= 4.1 \quad (\text{Ans})$$

π -coordinate of particle is increasing at a rate of 0.4 units per second.

(6)

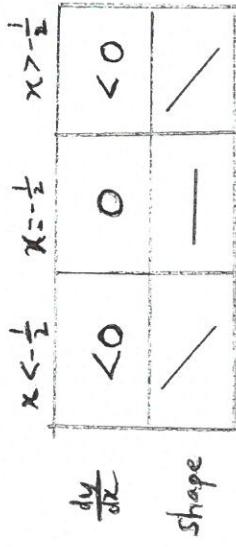
$$86(i) \quad \frac{dy}{dx} = 0$$

$$\therefore -3(2x+1)^2(2) = 0$$

$$(2x+1)^2 = 0$$

$$2x+1 = 0$$

$$x = -\frac{1}{2}$$



$$\text{When } x = -\frac{1}{2}, y = 8 - [2(-\frac{1}{2}) + 1]^3 \\ = 8$$

- There is only 1 stationary point $(-\frac{1}{2}, 8)$ which is a point of inflection, as indicated in the table above.

(7)



$$\begin{aligned} q_1) \quad m_{AB} &= \frac{p-1}{p+2} \\ &= \frac{p-1}{2} \\ m_{CB} &= \frac{p-3}{p-1} \\ &= 3-p \end{aligned}$$

Since $\angle ABO = \angle CBO$,

$$\begin{aligned} \tan \angle ABO &= \frac{1}{m_{AB}} = \frac{2}{p-1} \\ \tan \angle CBO &= \frac{1-0}{p-3} = \frac{1}{p-3} \\ \therefore \frac{2}{p-1} &= \frac{1}{p-3} \\ 2p-6 &= p-1 \end{aligned}$$

$p = 5$ (shown)

$$\begin{aligned} q_1) \quad m_{AB} &= \frac{p-1}{2} \\ &= 2 \\ m_{AD} &= -\frac{1}{2} \\ m_{CD} &= m_{AB} = 2 \\ \text{Eqn of AD: } y-1 &= -\frac{1}{2}(x+2) \\ y &= -\frac{1}{2}x + 1 \quad (1) \\ \text{Eqn of CD: } y-3 &= 2(x-1) \\ y &= 2x-1 \quad (2) \\ \text{Solve (1) into (2): } -\frac{1}{2}x &= 2x-1 \\ 2\frac{1}{2}x &= -1 \\ x &= -0.4 \\ \therefore y &= -\frac{1}{2}(-0.4) \\ &= 0.2 \end{aligned}$$

(8)

$$\text{i)} \quad \text{Area of } ABCD = \frac{1}{2} \begin{vmatrix} 0 & -2 & -0.4 & 1 & 0 \\ 5 & 1 & 0.2 & 3 & 5 \end{vmatrix}$$

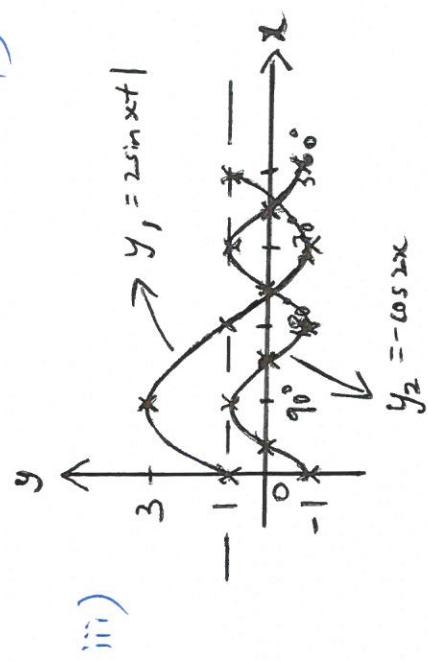
$$= \frac{1}{2} (0 - 0.4 - 1.2 + 5 + 10 + 0.4 - 0.2 - 0)$$

$$= 6.8 \text{ units}^2$$

a) $y_1 : \text{Amplitude} = 2$
 $\text{period} = 360^\circ$

b) $y_2 : \text{Amplitude} = 1$
 $\text{period} = 180^\circ$

iv) $y_1 - y_2 > 0$
 $y_1 > y_2$
 $\therefore x < 218.2^\circ \quad \underline{\text{or}} \quad x > 321.8^\circ$



$$\text{y}_2 = -\cos 2x$$

$$\text{y}_1 = 2 \sin x t$$

$$ii) \quad 4.90x = 90^\circ - \theta$$

Let shortest dist. of P from OX & OY be x km &
y km respectively.

$$\therefore \sin(90^\circ - \theta) = \frac{x}{4}$$

$$x = 4 \sin(90^\circ - \theta)$$

$$= 4 \cos \theta$$

$$\therefore \sin \theta = \frac{y}{4}$$

$$y = 4 \sin \theta$$

$$i) \quad \frac{1}{2} \times 3 \times 5 = \frac{1}{2} (3)(4) \sin \theta + \frac{1}{2} (4)(5) \sin(90^\circ - \theta)$$

$$\frac{15}{2} = 6 \sin \theta + 10 \cos \theta$$

$$20 \cos \theta + 12 \sin \theta = 15 \quad (\text{shown})$$

$$ii) \quad R = \sqrt{20^2 + 12^2}$$

$$= \sqrt{544}$$

$$\tan \alpha = \frac{12}{20}$$

$$\alpha = 30.964^\circ \quad (\text{s.f.})$$

$$\therefore 20 \cos \theta + 12 \sin \theta = \sqrt{544} \cos(\theta - 31.0^\circ)$$

$$iv) \quad \therefore \sqrt{544} \cos(\theta - 31.0^\circ) = 15$$

$$\cos(\theta - 31.0^\circ) = \frac{15}{\sqrt{544}}$$

$$\alpha = 49.975^\circ \quad (\text{s.f.})$$

$$\theta - 30.964^\circ = 49.975^\circ$$

$$\theta = 80.9^\circ \quad (1 d.p.)$$