2020 A Math 4047/01 Answer Key

Qn 1.

$2x^2-5x+8=0$

$a=2,\ b=-5,\ c=8$

$\alpha+\beta=\frac{-b}{a}$

$=\frac{-(-5)}{2}$

$= \cancel{25}\,\frac{5}{2}.$

$\alpha\beta=\frac{c}{a}=\frac{8}{2}$

$=4.$

$\text{S.O.R.}=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$=\frac{\alpha^2+\beta^2}{\alpha\beta}$

$=\frac{\alpha^2+2\alpha\beta+\beta^2-2\alpha\beta}{\alpha\beta}$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$=\frac{\left(\frac{5}{2}\right)^2-2(4)}{4}$

$=\frac{\frac{25}{4}-8}{4}$

$=-\frac{7}{16}$

$\text{P.O.R.}=\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}$

$=1.$

eqn: $x^2-(\text{sor})\,x+(\text{por})=0$

$x^2-\left(-\frac{7}{16}\right)x+1=0$

$x^2+\frac{7}{16}x+1=0$

$16x^2+7x+16=0$ //.

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Qn 2

a) $$\left[\left(\frac{50}{3}\right)^{-2} \times \sqrt{3^3}\right] \div \frac{5}{2} = \left[\left(\frac{5\times5\times2}{3}\right)^{-2} \times 3^{\frac{3}{2}}\right] \times \frac{2}{5}$$

$$= \frac{5^{2\times(-2)} \times 2^{-2}}{3^{-2}} \times 3^{\frac{3}{2}} \times 2 \times 5^{-1}$$

$$= 5^{-4} \times 2^{-2} \times 3^{2} \times 3^{\frac{3}{2}} \times 2 \times 5^{-1}$$

$$= 2^{-2+1} \times 3^{2+\frac{3}{2}} \times 5^{-4-1}$$

$$= 2^{-1} \times 3^{\frac{7}{2}} \times 5^{-5}$$

$\therefore a=-1,\ b=3.5,\ c=-5.$

b) 2014 value $= 10^6$

2015 value $= 10^6 \times \frac{107}{100} = 10^6 \times 1.07$

2016 value $= 10^6 \times \frac{107}{100} \times \frac{107}{100} = 10^6 \times 1.07^2$

$\vdots$

2020 value $= 10^6 \times 1.07^6$

$= 10.7^6$

$= \$1500730$

$\approx \$1500\,000$ (2sf).

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Qn 3

$$\begin{array}{r} 2 \\ 2x^2-x-3\,\overline{)\,4x^2-7x+9} \\ 4x^2-2x-6 \\ \hline -5x+15. \end{array} \qquad \therefore \frac{4x^2-7x+9}{2x^2-x-3} = 2 + \frac{-5x+15}{2x^2-x-3.}$$

$$\frac{-5x+15}{2x^2-x-3} = \frac{-5x+15}{(2x-3)(x+1)} = \frac{A}{2x-3} + \frac{B}{x+1}$$

$\times(2x-3)(x+1)$, $\quad -5x+15 = A(x+1) + B(2x-3)$

when $x=-1$, $\quad -5(-1)+15 = A(0) + B(-2-3)$

$$20 = B(-5)$$

$$\therefore B = -4.$$

when $x = \frac{3}{2}$, $\quad -5\left(\frac{3}{2}\right)+15 = A\left(\frac{3}{2}+1\right) + B(0)$

$$\frac{15}{2} = A\left(\frac{5}{2}\right)$$

$$\therefore A = 3.$$

$$\therefore \frac{4x^2-7x+9}{2x^2-x-3} = 2 + \frac{-5x+15}{(2x-3)(x+1)}$$

$$= 2 + \frac{3}{2x-3} - \frac{4}{x+1.}$$

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Qn 4

$$y = \frac{2x-3}{x^2+4}.$$

$$\frac{dy}{dx} = \frac{2(x^2+4) - 2x(2x-3)}{(x^2+4)^2}$$ using quotient rule.

$$= \frac{2x^2+8-4x^2+6x}{(x^2+4)^2}$$

$$= \frac{-2(x^2-3x-4)}{(x^2+4)^2}$$

$$= \frac{-2(x-4)(x+1)}{(x^2+4)^2}.$$

for y to be increasing,

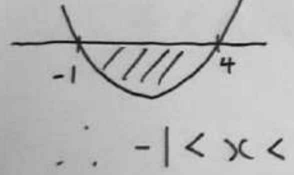
$$\frac{dy}{dx} > 0.$$

$$\therefore \frac{-2(x-4)(x+1)}{(x^2+4)^2} > 0$$

$$\frac{(x-4)(x+1)}{(x^2+4)^2} < 0$$

as $(x^2+4)^2 > 0$,

$$(x-4)(x+1) < 0.$$



$$\therefore -1 < x < 4_{//}.$$

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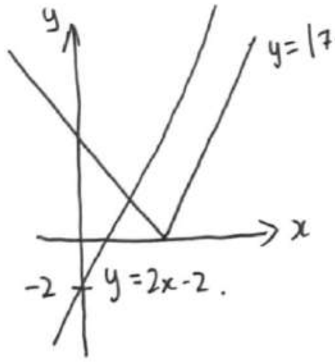
Qn5

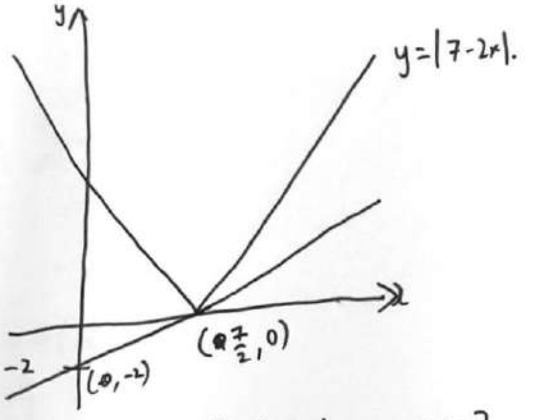
consider when $m = 2$, .



when $m=2$,
line $y=mx-2$ is parallel
to right side of $y=|7-2x|$.
∴ it will only cut at one point.

when $m>2$,
$y=mx-2$ diverges from right side
of $y=|7-2x|$ and ∴ will
always cut at only one point.

now,
consider the following:



when m is such that line $y=mx-2$
touches the minimum point of $y=|7-2x|$,
it will cut only at one point.

∴ when $y=0$,

$$|7-2x|=0$$
$$7-2x=0$$
$$2x=7$$
$$x=\frac{7}{2}.$$

∴ grad of m at this pt

$$=\frac{y_1-y_2}{x_1-x_2}=\frac{0-(-2)}{\frac{7}{2}-0}$$

$$=\frac{4}{7}.$$

~~when m > $\frac{4}{7}$, it will no~~
when $m<\frac{4}{7}$, it will no longer
cut the $y=|7-2x|$ graph.

∴ Based on the above,

$$\frac{4}{7}<m<2.$$

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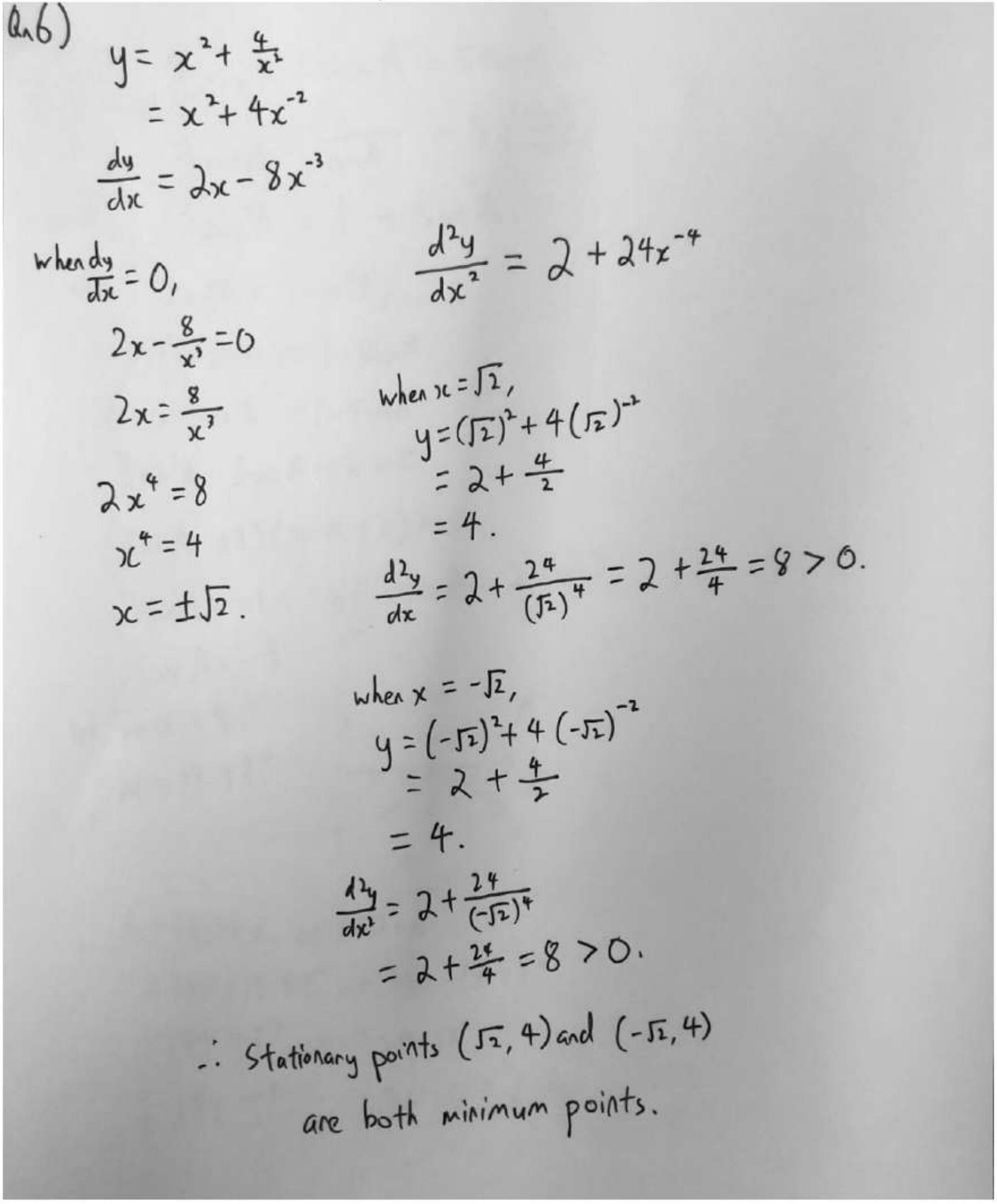


Qn6)

$y = x^2 + \frac{4}{x^2}$

$= x^2 + 4x^{-2}$

$\frac{dy}{dx} = 2x - 8x^{-3}$

when $\frac{dy}{dx} = 0$,

$2x - \frac{8}{x^3} = 0$

$2x = \frac{8}{x^3}$

$2x^4 = 8$

$x^4 = 4$

$x = \pm\sqrt{2}$.

$\frac{d^2y}{dx^2} = 2 + 24x^{-4}$

when $x = \sqrt{2}$,

$y = (\sqrt{2})^2 + 4(\sqrt{2})^{-2}$

$= 2 + \frac{4}{2}$

$= 4$.

$\frac{d^2y}{dx} = 2 + \frac{24}{(\sqrt{2})^4} = 2 + \frac{24}{4} = 8 > 0$.

when $x = -\sqrt{2}$,

$y = (-\sqrt{2})^2 + 4(-\sqrt{2})^{-2}$

$= 2 + \frac{4}{2}$

$= 4$.

$\frac{d^2y}{dx^2} = 2 + \frac{24}{(-\sqrt{2})^4}$

$= 2 + \frac{24}{4} = 8 > 0$.

$\therefore$ Stationary points $(\sqrt{2}, 4)$ and $(-\sqrt{2}, 4)$ are both minimum points.

Qn 7

$$3\cos A = \sec A - 5\tan A.$$

$$3\cos A = \frac{1}{\cos A} - 5\left(\frac{\sin A}{\cos A}\right).$$

$\times\cos A,\quad 3\cos^2 A = 1 - 5\sin A.$

sub $\cos^2 A = 1 - \sin^2 A,$

$$3(1-\sin^2 A) = 1 - 5\sin A$$

$$3 - 3\sin^2 A = 1 - 5\sin A$$

$$3\sin^2 A - 5\sin A - 2 = 0.$$

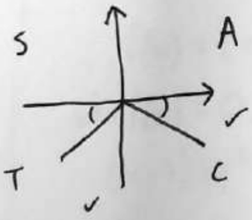
$$(3\sin A + 1)(\sin A - 2) = 0.$$

$\therefore 3\sin A = -1$ or $\sin A = 2$ (rej).

$\therefore \sin A = -\frac{1}{3}.$

let $\sin\alpha = \frac{1}{3}$

$\alpha = 19.47°$



$\therefore A = 180 + \alpha$ or $360 - \alpha$

$= 180 + 19.47°$ or $360 - 19.47°$

$= 199.47°$ or $340.53°$

$\approx 199.5°$ or $340.5°$ (1dp).

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Qn 8

a) Let $f(x) = x^3 + ax$.

$f(2) = 2^3 + a(2)$

$= 8 + 2a$.

$f(-1) = (-1)^3 + a(-1)$

$= -1 - a$.

Since remainder is the same,

$f(2) = f(-1)$

$8 + 2a = -1 - a$

$3a = -9$

$a = -3$ //.

b) Let $f(x) = x^3 - 2x^2 - 4x + 3$.

$f(3) = 3^3 - 2(3)^2 - 4(3) + 3$

$= 27 - 18 - 12 + 3$

$= 0$.

$\therefore (x-3)$ is a root.

$$
\begin{array}{r|l}
 & x^2 + x - 1 \\
\hline
x-3 & x^3 - 2x^2 - 4x + 3 \\
 & x^3 - 3x^2 \\
\hline
 & \quad x^2 - 4x \\
 & \quad x^2 - 3x \\
\hline
 & \qquad -x + 3 \\
 & \qquad -x + 3 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$\therefore f(x) = (x-3)(x^2 + x - 1)$.

when $f(x) = 0$,

$(x-3)(x^2 + x - 1) = 0$.

$x = 3$ or $x^2 + x - 1 = 0$

$x = 3$ or $x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{5}}{2}$

$\therefore x = 3, \dfrac{-1+\sqrt{5}}{2}, \dfrac{-1-\sqrt{5}}{2}$ //.

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Qn 9

i) $x^2+y^2+4x-6y-12=0$

$x^2+4x+y^2-6y=12.$

$x^2+4x+4-4+y^2-6y+9-9=12.$

$(x+2)^2+(y-3)^2-4-9=12.$

$\therefore (x+2)^2+(y-3)^2=25=5^2.$

$\therefore$ centre $= (-2, 3)$

radius $= 5$ //.

ii) The normal to the tangent at any point of a circle will pass through the centre.

$\therefore$ grad of normal to tangent

$$= \frac{y_1-y_2}{x_1-x_2} = \frac{7-3}{1-(-2)}$$

$$= \frac{4}{3}$$

$$\text{grad of tangent} = -\frac{1}{\frac{4}{3}}$$

$$= -\frac{3}{4}.$$

eqn of tangent : $y-7=-\frac{3}{4}(x-1)$

$y=-\frac{3}{4}x+\frac{3}{4}+7$

$4y=-3x+31$ //.

Qn 10).

i) $y = \cos 2x$

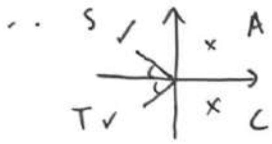
$y = -\frac{\sqrt{3}}{2}$.

$\cos 2x = -\frac{\sqrt{3}}{2}$.

Let $\cos\alpha = \frac{\sqrt{3}}{2}$

$\alpha = \frac{\pi}{6}$ (crit angle).

∴



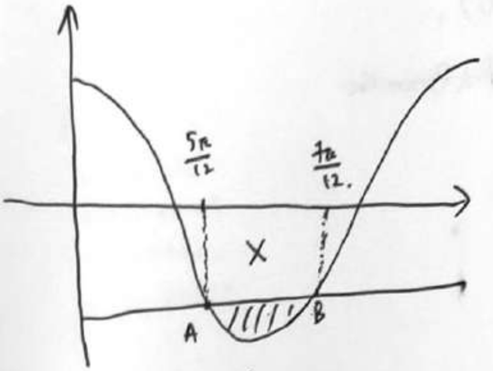
$2x = \pi - \frac{\pi}{6}$ or $\pi + \frac{\pi}{6}$

$x = \frac{5\pi}{12}$ or $\frac{7\pi}{12}$.

∴ $A = \frac{5\pi}{12}$

$B = \frac{7\pi}{12}$.

ii)



$\text{Area}X = \frac{\sqrt{3}}{2} \times \left(\frac{7\pi}{12} - \frac{5\pi}{12}\right)$

$= \frac{\sqrt{3}}{2} \times \frac{\pi}{6}$

$= \frac{\pi\sqrt{3}}{12}$.

Area above graph from A to B $= -\int_{\frac{5\pi}{12}}^{\frac{7\pi}{12}} \cos 2x \, dx$

$= -\left[\frac{1}{2}\sin 2x\right]_{\frac{5\pi}{12}}^{\frac{7\pi}{12}}$

$= -\left[\frac{1}{2}\sin\frac{7\pi}{6} - \frac{1}{2}\sin\frac{5\pi}{6}\right]$

$= -\left[\frac{1}{2}(-\frac{1}{2}) - \frac{1}{2}(\frac{1}{2})\right]$

$= -\left(-\frac{1}{2}\right) = \frac{1}{2}$.

Shaded area = Area above graph from A to B − Area X.

$= \frac{1}{2} - \frac{\pi\sqrt{3}}{12}$

$= \frac{6 - \pi\sqrt{3}}{12}$ //.

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Qn 11.

$V = 15\left(\frac{t}{20} + e^{kt}\right)$

$= \frac{15}{20}t + 15e^{kt}.$

* the value of k can be left in decimal as well. in which case, $k = 0.04054$.

i) At A, $t = 0$.

$\therefore V = 0 + 15e^{0}$

$= 15 \text{ m/s}$.

$\therefore$ At B,

$V = 2 \times 15$

$= 30 \text{ m/s}$ //.

ii) when $t = 10$, $v = 30$.

$\therefore 30 = 15\left(\frac{10}{20} + e^{10k}\right)$

$2 = \frac{1}{2} + e^{10k}$

$e^{10k} = \frac{3}{2}$

$10k = \ln\frac{3}{2}$

$k = \frac{1}{10}\ln\frac{3}{2}$

$d = \int \frac{15}{20}t + 15e^{kt}\,dt.$

$= \frac{15}{40}t^2 + \frac{15}{k}e^{kt} + c.$

when $t = 0$, $d = 0$.

$\therefore 0 = \frac{15}{40}(0) + \frac{15}{k}e^{0} + c$

$\therefore c = -\frac{15}{k}.$

when $t = 10$, $k = \frac{1}{10}\ln\frac{3}{2}$,

$d = \frac{15}{40}(10)^2 + \frac{15e^{\frac{10}{10}\ln\frac{3}{2}}}{\frac{1}{10}\ln\frac{3}{2}} - \frac{15}{\frac{1}{10}\ln\frac{3}{2}}$

$= 37.5 + 184.96$

$= 222.466$

$\approx 222 \text{ m}$ (3sf).

iii) $a = \frac{dv}{dt} = \frac{15}{20} + 15ke^{kt}.$

when $t = 2$,

$a = \frac{15}{20} + 15\left(\frac{1}{10}\ln\frac{3}{2}\right)e^{\frac{2}{10}\ln\frac{3}{2}}$

$= 1.4094$

$\approx 1.41 \text{ m/s}^2$ //.

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Qn 12

i) $\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$

$= \dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta+\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta+\cos\alpha\cos\beta+\sin\alpha\sin\beta}$

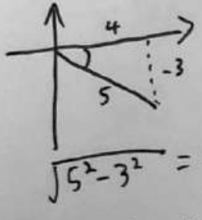
$= \dfrac{2\sin\alpha\cos\beta}{2\cos\alpha\cos\beta}$

$= \dfrac{\sin\alpha}{\cos\alpha}$

$= \tan\alpha =$ RHS. (shown).

ii) since A & B are both acute,
A-B is also acute.
$\therefore$ if $\sin(A-B) < 0$
$A-B < 0$
$A < B$
B is larger than A.

iii) $\sin(A-B) = -\frac{3}{5}$
since A-B is acute,
& $\sin(A-B) < 0$,
$\sin(A-B)$ belongs in 4th Quad.



$\sqrt{5^2-3^2} = 4.$

$\therefore \cos(A-B) = \dfrac{\text{adj}}{\text{hyp}}$
$= \dfrac{4}{5}$ //.

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Qn 12

iv) $\sin(A+B) = \tan(A+B) \times \cos(A+B)$

$= \frac{24}{7} \times \frac{7}{25}$

$= \frac{24}{25}$ //.

v) $$\tan A = \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)}$$

$$= \frac{\frac{24}{25} + \left(-\frac{3}{5}\right)}{\frac{7}{25} + \frac{4}{5}} = \frac{\frac{9}{25}}{\frac{27}{25}} = \frac{9}{27} = \frac{1}{3} \text{ (shown).}$$

vi) $$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{24}{7} = \frac{\frac{1}{3} + \tan B}{1 - \frac{1}{3}\tan B}$$

$\frac{24}{7}\left(1 - \frac{1}{3}\tan B\right) = \frac{1}{3} + \tan B$

$\frac{24}{7} - \frac{8}{7}\tan B = \frac{1}{3} + \tan B$

$\frac{8}{7}\tan B + \tan B = \frac{24}{7} - \frac{1}{3}$

$\frac{15}{7}\tan B = \frac{65}{21}$

$\tan B = \frac{13}{9}$ //.