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2020 A Math 4047/02 Answer Key

Qn 1)

i) $7\cos\theta + 4\sin\theta$

$a = 7,\ b = 4.$

$R = \sqrt{7^2 + 4^2} = \sqrt{65}$

$\alpha = \tan^{-1}\frac{4}{7}$

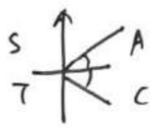
$= 0.5191$

$7\cos\theta + 4\sin\theta = 6$

$\sqrt{65}\cos(\theta - 0.5191) = 6$

$\cos(\theta - 0.5191) = \frac{6}{\sqrt{65}}$

$\cos A = \frac{6}{\sqrt{65}}$



$A = 0.7314$ rad

$\therefore \theta - 0.5191 = 0.7314$ or $\theta - 0.5191 = 2\pi - 0.7314$

$\theta = 1.2505$ or 6.070 (rej)

$\simeq 1.25$ (3sf).

ii) $80 - (7\cos\theta + 4\sin\theta)^2 = 80 - [\sqrt{65}\cos(\theta - 0.5191)]^2$

$= 80 - 65\cos^2(\theta - 0.5191).$

since $-1 \leq \cos(\theta - 0.5191) \leq 1$

$0 \leq \cos^2(\theta - 0.5191) \leq 1$

$-65 \leq -65\cos^2(\theta - 0.5191) \leq 0$

$15 \leq 80 - 65\cos^2(\theta - 0.5191) \leq 80$

$\therefore$ Max value $= 80$, $\cos^2(\theta - 0.5191) = 0$

$\cos(\theta - 0.5191) = 0$

$\theta - 0.5191 = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ (rej)

$\theta = \frac{\pi}{2} + 0.5191$

$= 2.089$

≈ 2.09 rad (3sf)

Min Value $= 15$.

$\cos^2(\theta - 0.5191) = 1$

$\cos(\theta - 0.5191) = 1$

$\theta - 0.5191 = 0$ or 2π (rej)

$\theta = 0.5191$ rad

$\simeq 0.519$ rad (3sf)

OR

$\cos(\theta - 0.5191) = -1$

$\theta - 0.5191 = \pi$

$\theta = \pi + 0.5191$ (rej)

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Qn 2)

a) $y = 2x^2 - 7$ — (1)
$y = 3x + 20$ — (2)

sub (1) into (2),

$2x^2 - 7 = 3x + 20$

$2x^2 - 3x - 27 = 0$

$(x+3)(2x-9) = 0$

$\therefore x = -3$ or $x = 4.5$

when $x = -3$, $y = 3(-3) + 20 = 11$

when $x = \frac{9}{2}$, $y = 3(\frac{9}{2}) + 20 = \frac{67}{2}$.

$\therefore x = -3, y = 11$
& $x = 4.5, y = 33.5$ //.

b) $f(x) = ax^2 + 5x - 2$
for $f(x)$ to always be negative,
(1) coefficient of $x^2 < 0$.
$\therefore a < 0$.

(2) $b^2 - 4ac < 0$
$5^2 - 4(a)(-2) < 0$
$25 + 8a < 0$
$a < -\frac{25}{8}$
$\therefore$ greatest integer $a = -4$ //.

c) $y = 4x + c$ — (1)
$y = x^2 + cx + \frac{21}{4}$. — (2)

sub (1) into (2),

$4x + c = x^2 + cx + \frac{21}{4}$

$x^2 + cx - 4x + \frac{21}{4} - c = 0$

$a = 1$, $b = c - 4$, $c = \frac{21}{4} - c$.

For line to be tangent,
there is only one real root.

$\therefore b^2 - 4ac = 0$
$(c-4)^2 - 4(1)(\frac{21}{4} - c) = 0$
$c^2 - 8c + 16 - 21 + 4c = 0$
$c^2 - 4c - 5 = 0$
$(c-5)(c+1) = 0$

$\therefore c = -1$ or $c = 5$ //

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Qn 3)

i) $\left(\frac{3}{x^2}+x\right)^8$

$T_{r+1} = \binom{8}{r}\left(\frac{3}{x^2}\right)^{8-r}(x)^r$

$= \binom{8}{r} . 3^{8-r} . x^{-2(8-r)} . x^r$

$= \binom{8}{r} . 3^{8-r} . x^{-16+3r}$.

for a constant term,

$-16+3r = 0$

$3r = 16$

$r = \frac{16}{3}$

as r is not an integer,
there is no constant term
and every term will be dependant on x.

ii) $\left(\frac{3}{x^2}+x\right)^8(5-2x)$

$\therefore$ Term independant
$=$ constant term of $\left(\frac{3}{x^2}+x\right)^8 \times 5$
$+$ coefficient of x^{-1} term $\times(-2)$

~~As there is no constant term,~~
~~when x^0,~~

for x^{-1} term,

$-16+3r = -1$

$3r = 15$

$r = 5$.

$\therefore T_{5+1} = \binom{8}{5} 3^3 . x^{-1}$

$= 1512x^{-1}$

$\therefore$ Independant term $= 0\times5 + 1512\times(-2)$

$= -3024$ //.

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Qn 5)

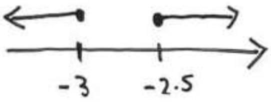
a) $15(1+2x) \geq x(19-2x)$

$15+30x \geq 19x-2x^2$

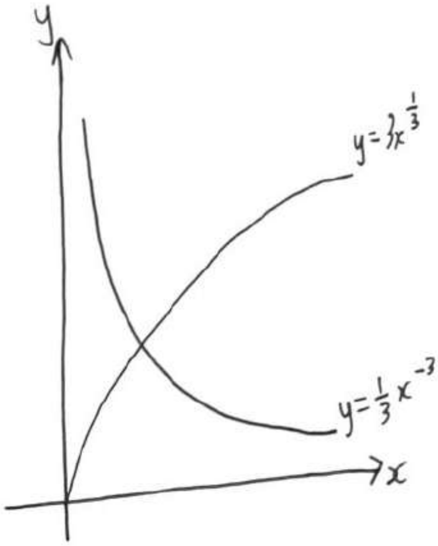
$2x^2+11x+15 \geq 0$

$(2x+5)(x+3) \geq 0$

$\therefore x \leq -3, \; x \geq -2.5$



b) i)



b) ii) $y=3x^{\frac{1}{3}}$ — ①

$y=\frac{1}{3}x^{-3}$ — ②

sub ① into ②, $3x^{\frac{1}{3}}=\frac{1}{3}x^{-3}$

$x^{\frac{1}{3}+3}=\frac{1}{9}$

$x^{\frac{10}{3}}=\frac{1}{9}$

$\left(x^{\frac{10}{3}}\right)^3=\left(\frac{1}{9}\right)^3$

$x^{10}=\frac{1}{729}$ (shown).

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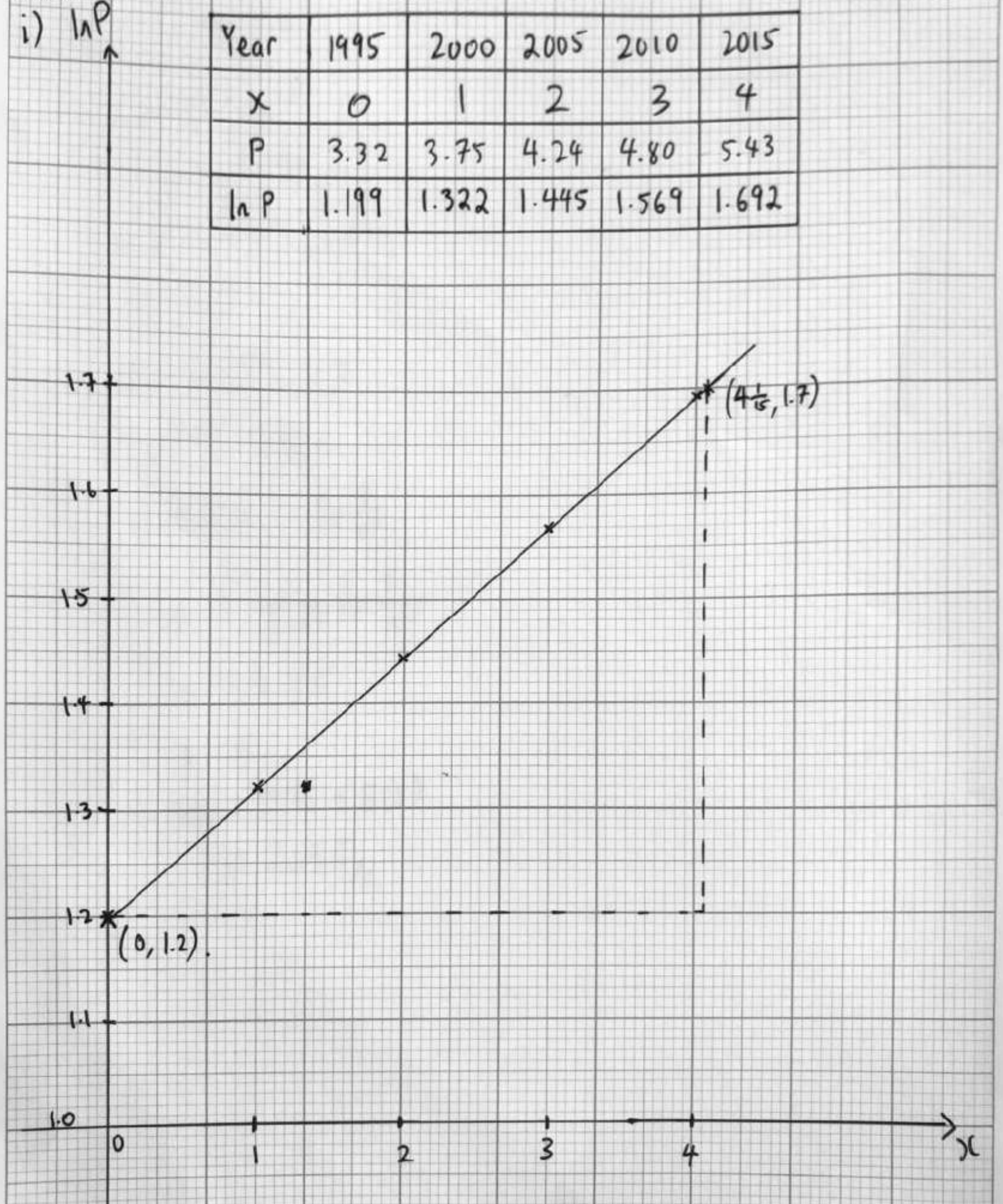
Qn6)

i)

Year	1995	2000	2005	2010	2015
x	0	1	2	3	4
P	3.32	3.75	4.24	4.80	5.43
ln P	1.199	1.322	1.445	1.569	1.692



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(Qn 6)

ii) gradient $= \frac{1.7-1.2}{4\frac{1}{15}-0}$

$= 0.1229$

$\ln P = 0.1229x + 1.2$

$P = e^{0.1229x+1.2}$

$= e^{0.1229x} \cdot e^{1.2}$

$= e^{1.2} \cdot e^{0.1229x}$

$\approx e^{1.2} \cdot e^{0.123x}$ (3 s.f.) //

iii) $P > 8$

$e^{1.2} \cdot e^{0.1229x} > 8$

$e^{0.1229x} > 8/e^{1.2}$

$0.1229x > \ln \frac{8}{e^{1.2}}$

$x > \frac{1}{0.1229} \ln \frac{8}{e^{1.2}}$

$x > 7.155$

$\therefore x = 8.$

$\therefore$ Year $= 1995 + 8(5)$

$= 2035$ //

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Qn 7)

i) $\frac{d}{dx}\{x(3x-5)^{\frac{5}{3}}\}$

$= (1)(3x-5)^{\frac{5}{3}} + (\frac{5}{3})(3x-5)^{\frac{2}{3}}(3)(x)$

$= (3x-5)^{\frac{5}{3}} + 5x(3x-5)^{\frac{2}{3}}$

$= (3x-5)^{\frac{2}{3}}(3x-5+5x)$

$= (8x-5)(3x-5)^{\frac{2}{3}}$ (shown).

ii) $\frac{d}{dx}\{x(3x-5)^{\frac{5}{3}}\} = 8x(3x-5)^{\frac{2}{3}} - 5(3x-5)^{\frac{2}{3}}$

$\int \frac{d}{dx}\{x(3x-5)^{\frac{5}{3}}\}dx = \int 8x(3x-5)^{\frac{2}{3}}dx - \int 5(3x-5)^{\frac{2}{3}}dx$

$\therefore \int 8x(3x-5)^{\frac{2}{3}}dx = x(3x-5)^{\frac{5}{3}} + \int 5(3x-5)^{\frac{2}{3}}dx$

$= \frac{1}{8}[x(3x-5)^{\frac{5}{3}} + (\frac{3}{5})(5)(3x-5)^{\frac{5}{3}}(3)] + c.$

$= \frac{1}{8}[x(3x-5)^{\frac{5}{3}} + (3x-5)^{\frac{5}{3}}] + c$

$= \frac{1}{8}(x+1)(3x-5)^{\frac{5}{3}} + C.$

iii) $\int_{-1}^{5/3} x(3x-5)^{\frac{2}{3}}dx = \left[\frac{1}{8}(x+1)(3x-5)^{5/3}\right]_{-1}^{5/3}$

$= \frac{1}{8}(\frac{5}{3}+1)(3(\frac{5}{3})-5)^{5/3} - \frac{1}{8}(-1+1)(3(-1)-5)^{5/3}$

$= 0 - 0 = 0.$

The result implies that the curve $y = x(3x-5)^{\frac{2}{3}}$ cuts the x-axis at least once from $x=-1$ to $x=\frac{5}{3}$.

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Qn8)

a) $e^{x}(1+e^{x})=\frac{3}{4}$

let $y=e^{x}$

$y(1+y)=\frac{3}{4}$

$y+y^{2}=\frac{3}{4}$

$4y^{2}+4y-3=0$

$\cancel{(2y+1)}\ (2y-1)(2y+3)=0$

$y=\frac{1}{2}$ or $y=-\frac{3}{2}$

$e^{x}=\frac{1}{2}$ or $e^{x}=-\frac{3}{2}$ (rej)

$\therefore x=\ln\frac{1}{2}$ //

b) $1+\log_2 y+\frac{1}{\log_8 2}=\log_2(y+3)$

$\log_2 2+\log_2 y+\log_2 8=\log_2(y+3)$

$\log_2(2\times y\times 8)=\log_2(y+3)$

$16y=y+3$

$y=\frac{3}{15}$

$=\frac{1}{5}$ //

c) $x=\ln\left(\frac{2x+7}{3}\right)^{2}$

$x=2\ln\left(\frac{2x+7}{3}\right)$

$\frac{x}{2}=\ln\left(\frac{2x+7}{3}\right)$

$e^{\frac{x}{2}}=\frac{2x+7}{3}$

$3e^{\frac{x}{2}}=2x+7$

$3e^{\frac{x}{2}}+4=2x+11$.

$\therefore$ line: $y=2x+11$ //

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Qn9)

i) $AB = \sqrt{[7-(-5)]^2+(5-0)^2}$
$= \sqrt{12^2+5^2}$
$= 13$ units.

Since $AD+AB = \frac{1}{2}$ perimeter,
$AD+AB = \frac{1}{2}(46)$
$= 23.$
$\therefore AD = 23-13$
$= 10$ units.

grad $BD = \frac{6-5}{5-7}$
$= -\frac{1}{2}.$

eqn BD: $y-5 = -\frac{1}{2}(x-7)$
$y = -\frac{1}{2}x + \frac{7}{2} + 5$
$= -\frac{1}{2}x + \frac{17}{2}$

If x value of $D = x$,
y value of $D = \frac{17-x}{2}.$
$\therefore D = \left(x, \frac{17-x}{2}\right)$

Since $AD = 10$,
$\sqrt{[x-(-5)]^2 + \left(\frac{17-x}{2}-0\right)^2} = 10$
$(x+5)^2 + \frac{(17-x)^2}{4} = 100$
$4(x^2+10x+25) + 289 - 34x + x^2 = 400$
$5x^2 + 6x + 389 = 400$
$5x^2 + 6x = 11$ (shown).

ii) $5x^2+6x-11 = 0$
$(5x+11)(x-1) = 0$
$x = 1$ or $x = -\frac{11}{5}.$

As both values of x are valid roots, Diagram is necessary as we can see that x value of D is larger than O as D is to the right of the y-axis.
$\therefore x = 1.$
$y = -\frac{1}{2}(1) + \frac{17}{2} = 8.$
$\therefore D = (1, 8)$

iii) Since AB & DC are parallel,
x value of AB = x value of DC.
$7-(-5) = C_x - 1$
$C_x = 13.$
y value of AB = y value of DC.
$5-0 = C_y - 8$
$C_y = 8+5$
$= 13.$
$\therefore C = (13, 13)$

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Qn 10)

i) $f''(x) = 48x^2 + 2e^{2x-1}$

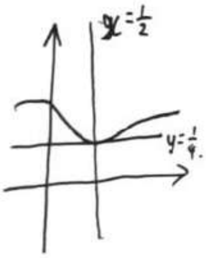
$f'(x) = \frac{48}{3}x^3 + \frac{2}{2}e^{2x-1} + C$

$= 16x^3 + e^{2x-1} + C.$

when $y = \frac{1}{4}$, $x = \frac{1}{2}$ is normal.

$\therefore y = \frac{1}{4}$ is tangent.

$\therefore$ gradient at $x = \frac{1}{2}$ is zero.



when $x = \frac{1}{2}$, $f'(x) = 0$.

$0 = 16\left(\frac{1}{2}\right)^3 + e^{2\left(\frac{1}{2}\right)-1} + C$

$C = -3.$

$\therefore f'(x) = 16x^3 + e^{2x-1} - 3$ //.

ii) $f(x) = \frac{16}{4}x^4 + \frac{1}{2}e^{2x-1} - 3x + D.$

$= 4x^4 + \frac{1}{2}e^{2x-1} - 3x + D.$

when $x = \frac{1}{2}$, $y = \frac{1}{4}$.

$\therefore \frac{1}{4} = 4\left(\frac{1}{2}\right)^4 + \frac{1}{2}e^{2\left(\frac{1}{2}\right)-1} - 3\left(\frac{1}{2}\right) + D$

$\frac{1}{4} = \frac{1}{4} + \frac{1}{2} - \frac{3}{2} + D$

$D = 1.$

$\therefore f(x) = 4x^4 + \frac{1}{2}e^{2x-1} - 3x + 1$ //.

iii) When curve intersects y axis,

$x = 0.$

$f(x) = 0 + \frac{1}{2}e^{-1} + 1$

$= \frac{1}{2e} + 1$

$= \frac{1+2e}{2e}.$

grad of tangent at $x = 0$,

$f'(x) = 0 + e^{-1} - 3$

$= \frac{1}{e} - 3$

$= \frac{1-3e}{e}.$

eqn when $x = 0$, $y = \frac{1+2e}{2e}$, grad $= \frac{1-3e}{e}$

$y - \frac{1+2e}{2e} = \frac{1-3e}{e}(x - 0)$

$\times 2e$, $2ey - 1 - 2e = (2 - 6e)x$

$2ey - 1 - 2e = 2x - 6ex$

$2ey + 6ex - 2e = 2x + 1$

$2e(y + 3x - 1) = 2x + 1$ (shown) //.

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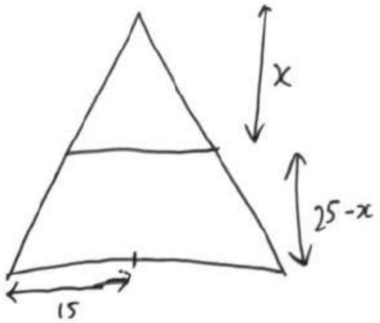
Qn 11)

i) $V_{water} = V_{big\,cone} - V_{small\,cone}.$



$$\frac{r_{small}}{r_{big}} = \frac{h_{small}}{h_{big}}$$

$$\frac{r_{small}}{15} = \frac{x}{25}$$

$$r_{small} = \frac{15}{25}x = \frac{3}{5}x.$$

$$V = \frac{1}{3}\pi(15)^2(25) - \frac{1}{3}\pi\left(\frac{3}{5}x\right)^2(x)$$

$$= \frac{1}{3}\pi\left[5625 - \frac{9}{25}x^3\right]$$

$$= \frac{9}{3}\pi\left[625 - \frac{1}{25}x^3\right]$$

$$= 3\pi\left[625 - \frac{x^3}{25}\right] \text{ (shown)} //.$$

ii) $V = 1875\pi - \frac{3\pi}{25}x^3$

$$\frac{dV}{dt} = -\frac{9\pi}{25}x^2$$

$$\frac{dV}{dx} = \frac{dV}{dt} \times \frac{dt}{dx}$$

$$\therefore \frac{dt}{dx} = \frac{dV}{dx} \div \frac{dV}{dt}$$

$$= -\frac{9\pi}{25}x^2 \div kx^2$$

$$= -\frac{9\pi}{25k} //.$$

iii) ~~when t~~ $\frac{dt}{dx} = -\frac{9\pi}{25k}$

$$t = \int -\frac{9\pi}{25k}\,dx$$

$$= -\frac{9\pi}{25k}x + c.$$

when $t = 0$, $x = 25$.

$$0 = -\frac{9\pi}{25k}(25) + c$$

$$c = \frac{9\pi}{k}.$$

when $t = 72\pi$, $h = 12$ $\therefore x = 25 - 12 = 13$.

$$72\pi = \frac{-9\pi}{25k}(13) + \frac{9\pi}{k}$$

$$8 = -\frac{13}{25k} + \frac{1}{k}$$

$$8k = -\frac{13}{25} + 1$$

$$k = 0.06 //.$$

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