Subject/Topic: Date:

1a) $2x^2-8x+11 = 2\left[x^2-4x+\frac{11}{2}\right]$

$= 2\left[(x-2)^2-(2)^2+\frac{11}{2}\right]$

$= 2(x-2)^2+3$

stationary point is $(2,3)$

b)

$2x^2-8x+11 = 2x+3$

$2x^2-10x+8=0$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

$\therefore x=4$ or $x=1$

when $x=4$, when $x=1$,

$y=2(4)+3$ $y=2(1)+3$

$=11$ $=5$

$|AB| = \sqrt{(4-1)^2+(11-5)^2}$

$\sqrt{k} = \sqrt{45}$

$k=45$

2b) when $t=0$

$\ln m=3$

$m=e^3$

$=20.086$

$=20.1g$ (3sf)

c) when $m=\frac{20.086}{2}$

$\ln m = 2.31$ (3sf)

$t=2.8$ min

$=2$ min 48 sec

3a)

$$\begin{array}{r} 2 \\ x^3+2x \,\big)\, 2x^3 + 5x^2 + 6 \\ -\,(2x^3 + 4x) \\ \hline 5x^2 - 4x + 6 \end{array}$$

$$\frac{2x^3+5x^2+6}{x^3+2x} = 2 + \frac{5x^2-4x+6}{x^3+2x}$$

b)

$$\frac{2x^3+5x^2+6}{x^3+2x} = 2 + \frac{5x^2-4x+6}{x(x^2+2)}$$

$$= 2 + \frac{A}{x} + \frac{Bx+C}{x^2+2}$$

$$5x^2 - 4x + 6 = A(x^2+2) + (Bx+C)(x)$$

when $x = 0$,

$$6 = 2A$$

$$A = 3$$

when $x = 1$,

$$5 - 4 + 6 = 3(3) + (B+C)$$

$$B + C = -2$$

$$B = -2 - C \quad -①$$

when $x = 2$,

$$5(2)^2 - 4(2) + 6 = 3(2^2+2) + (2)(2B+C)$$

$$18 = 18 + 4B + 2C$$

$$4B + 2C = 0$$

$$2B + C = 0 \quad -②$$

sub ① into ②,

$$2(-2-C) + C = 0$$

$$-4 - 2C + C = 0$$

$$-4 - C = 0$$

$$C = -4$$

$$B = -2 - (-4)$$

$$= 2$$

$$\therefore \frac{2x^3+5x^2+6}{x^3+2x} = 2 + \frac{3}{x} + \frac{2x-4}{x^2+2}$$

Subject/Topic: Date:

4a) when $x = k$,

$y = 14(k) - k^2$

$= 14k - k^2$

when $x = 2k$,

$y = 14(2k) - (2k)^2$

$= 28k - 4k^2$

$28k - 4k^2 > 14k - k^2$

$-3k^2 + 14k > 0$

$3k^2 - 14k < 0$

$k(3k - 14) < 0$

$\therefore \quad 0 < k < 4\frac{2}{3}$

b) $\frac{dy}{dx} = 14 - 2x$

since $\frac{dy}{dx} = 14 - 2x$ and $k > 0$, gradient ~~of~~ at A will always be greater than at B since x-coordinate of B is larger.

5a) $(2 - \frac{ax}{2})^5 = (2)^5 + {}^5C_1(2)^4(-\frac{ax}{2}) + {}^5C_2(2)^3(-\frac{ax}{2})^2 + {}^5C_3(2)^2(-\frac{ax}{2})^3 + \ldots$

$= 32 - 40ax + 20a^2x^2 - 5a^3x^3 + \ldots$

b) $(2)(20a^2) + (3)(-40a) = 0$

$40a^2 - 120a = 0$

$a^2 - 3a = 0$

$a(a-3) = 0$

$a = 0$ (rej) or $a = 3$

$\therefore \quad a = 3$

c) coefficient of $x^3 = (2)(-5)(3)^3 + (3)(20)(3)^2$

$= 270$ //

Subject/Topic: Date:

6) $\frac{d^2y}{dx^2} = 2e^{-x} + 6e^{2x}$

$\frac{dy}{dx} = \int 2e^{-x} + 6e^{2x}\,dx$

$= -2e^{-x} + \frac{6e^{2x}}{2} + c$

$= -2e^{-x} + 3e^{2x} + c$

when $\frac{dy}{dx} = 3$, $x = 0$

$3 = -2 + 3 + c$

$c = 2$

$\frac{dy}{dx} = -2e^{-x} + 3e^{2x} + 2$

$y = \int -2e^{-x} + 3e^{2x} + 2\,dx$

$= 2e^{-x} + \frac{3}{2}e^{2x} + 2x + c$

when $x = 0$, $y = 5$

$5 = 2 + \frac{3}{2} + c$

$c = 1\frac{1}{2}$

$\therefore\ y = 2e^{-x} + \frac{3e^{2x}}{2} + 2x + 1\frac{1}{2}$

7i) $\angle PQR = \angle RPS$ (alt segment theorem)

$\angle PRS = \angle PQR$ (alt segment theorem)

$\angle RST = \angle PRS + \angle RPS$ (ext $\angle$ of $\triangle$)

$= x + x$

$= 2x$ // (Proven)

ii) $\angle PRQ = 90°$ ($\angle$ in semi-circle)

$\angle PRT = 180° - 90°$ ($\angle$s in a straight line)

$= 90°$

$\angle SRT = 90° - x$

$\angle PTR = 180° - 90° - x$ (Sum of $\angle$ in $\triangle$)

$= 90° - x$

$\angle SRT = \angle PTR$, $\therefore$ $\triangle RST$ is isosceles //

Subject/Topic: Date:

8a) $y = 3\sqrt{x}$

$4y = 3x + 9$

$y = \frac{1}{4}(3x+9)$

$3\sqrt{x} = \frac{1}{4}(3x+9)$

$12\sqrt{x} = 3x+9$

$[12(\sqrt{x})]^2 = (3x+9)^2$

$144x = 9x^2 + 54x + 81$

$9x^2 - 90x + 81 = 0$

$x^2 - 10x + 9 = 0$

$(x-1)(x-9) = 0$

$x = 1$ or $x = 9$

when $x = 1$, $y = 3\sqrt{1} = 3$; when $x = 9$, $y = 3\sqrt{9} = 9$

$\therefore$ P is $(1,3)$ and Q is $(9,9)$

b) $y = 3\sqrt{x}$

$= 3x^{\frac{1}{2}}$

$\frac{dy}{dx} = (3)(\frac{1}{2})(x^{-\frac{1}{2}})$

$= \frac{3}{2\sqrt{x}}$

$\frac{3}{2\sqrt{x}} = \frac{3}{4}$

$2\sqrt{x} = 4$

$\sqrt{x} = 2$

$x = 4$

when $x = 4$,

$y = 3\sqrt{4}$

$= 3(2)$

$= 6$

when $x = 4$, $y = 6$,

$4(6) = 3(4) + k$

$k = 12$

9a) $y = x^3 - ax^2 + bx + 4$

$\frac{dy}{dx} = 3x^2 - 2ax + b$

$3x^2 - 2ax + b > 0$

$b^2 - 4ac < 0$

$(-2a)^2 - 4(3)(b) < 0$

$4a^2 - 12b < 0$

$4a^2 < 12b$

$a^2 < 3b$ // (shown)

b) $y = x^3 - 8x^2 + 10x + 4$

when $x = 2$,

$y = (2)^3 - 8(2)^2 + 10(2) + 4$

$= 0$

$\therefore$ by factor theorem, $(x-2)$ is a factor of $x^3 - 8x^2 + 10x + 4$

$x^3 - 8x^2 + 10x + 4 = (x-2)(x^2 - 6x - 2)$

$$\begin{array}{r} x^2 - 6x - 2 \\ x-2 \enclose{longdiv}{x^3 - 8x^2 + 10x + 4} \\ -(x^3 - 2x^2) \\ \hline -6x^2 + 10x \\ -(-6x^2 + 12x) \\ \hline -2x + 4 \\ -(-2x + 4) \\ \hline 0 \end{array}$$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$

$= \frac{6 \pm \sqrt{44}}{2}$

$x = \frac{6 + \sqrt{44}}{2}$ or $x = \frac{6 - \sqrt{44}}{2}$

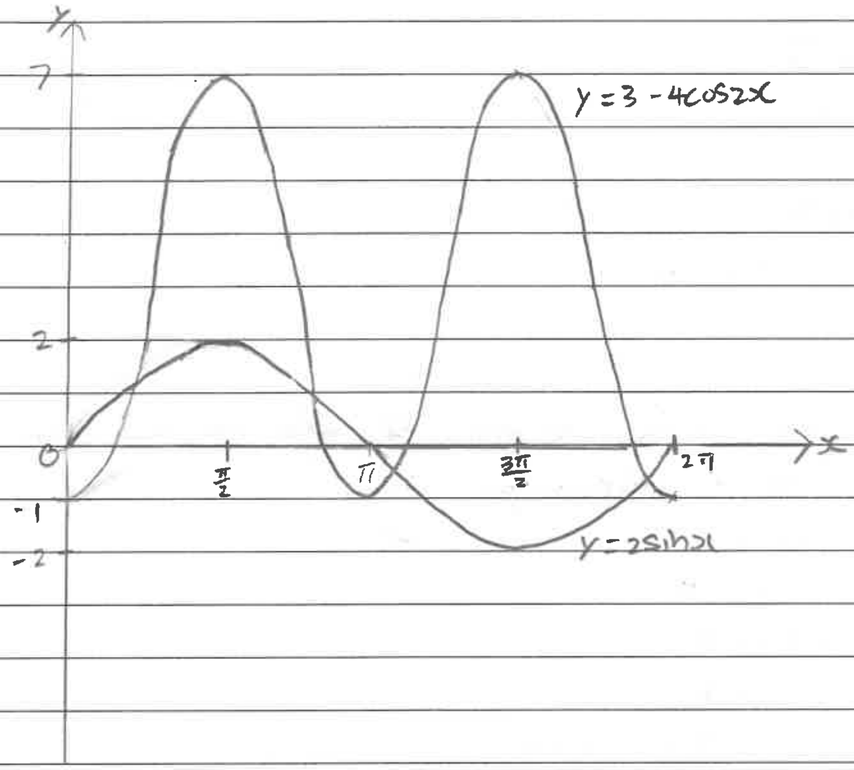
$= 3 + \sqrt{11}$ $= 3 - \sqrt{11}$

$\therefore$ the 3 x-coordinates are 2, $3 + \sqrt{11}$, $3 - \sqrt{11}$

Subject/Topic: Date:

10a i) amplitude $= 2$
period $= 2\pi$

ii) amplitude $= 4$
period $= \pi$



Subject/Topic: Date:

11(a) $\frac{1}{2} \times b \times h = 15$

$\frac{1}{2} \times (6)(q-2) = 15$

$q-2 = 5$

$q = 7$

b) $|AC| = \sqrt{89}$

$\sqrt{(4-p)^2 + (2-7)^2} = \sqrt{89}$

$16 - 8p + p^2 + 25 = 89$

$p^2 - 8p - 48 = 0$

$(p-12)(p+4) = 0$

$\therefore p = 12$ or $p = -4$

12a) $x = a\sin nt$

Since a is the amplitude of the sine graph of $a\sin nt$ and 60cm is the maximum value of displacement, $a = 60$

when $t = 6$, $x = 0$

$0 = 60\sin 6n$

$\sin 6n = 0$

$6n = 2\pi$

$n = \frac{1}{3}\pi$ (shown)

b) $x = 60\sin(\frac{t}{3}\pi)$

$v = \frac{dx}{dt}$

$= 60\cos(\frac{t}{3}\pi)(\frac{\pi}{3})$

$= 20\pi\cos(\frac{t}{3}\pi)$

when $t = 0$,

$\therefore$ max value of $v = 20\pi$ cm/s

c) $a = \frac{dv}{dt}$

$= -20\pi \sin\left(\frac{t}{3}\pi\right)\left(\frac{\pi}{3}\right)$

$= -\frac{20}{3}\pi^2 \sin\left(\frac{t}{3}\pi\right)$

when $x = 60$,

$\sin\left(\frac{t}{3}\pi\right) = 1$

$\frac{t}{3}\pi = \frac{\pi}{2}$

$t = \frac{3}{2}$

when $t = \frac{3}{2}$,

$a = -\frac{20}{3}\pi^2\left(\sin\frac{\pi}{2}\right)$

$= -65.797$

$\approx -65.8 \text{ cm/s}^2$ (3sf)

$\therefore$ magnitude of a is 65.8

13a) $y = \frac{4x+2}{x+1}$

$\frac{dy}{dx} = \frac{(x+1)(4) - (4x+2)(1)}{(x+1)^2}$

$= \frac{4x+4-4x-2}{(x+1)^2}$

$= \frac{2}{(x+1)^2}$

For $x > -1$, $\frac{dy}{dx} > 0$, $\therefore$ the curve does not have a stationary point

b) when $\frac{dy}{dx} = \frac{1}{2}$,

$\frac{2}{(x+1)^2} = \frac{1}{2}$

$(x+1)^2 = 4$

$x+1 = \pm 2$

$x = 2-1$ or $x = -2-1$

$= 1$ or $x = -3$ rej

when $x = 1$,

$y = \frac{4(1)+2}{1+1}$ $\therefore$ P is $(1, 3)$

$= \frac{6}{2} = 3$

gradient of normal $= -1 \div \frac{1}{2}$
$= -2$

$y - 3 = -2(x - 1)$
$y = -2x + 2 + 3$
$= -2x + 5$

when $x = 0$,
$y = 5$
when $y = 0$
$-2x = -5$
$x = \frac{5}{2}$

area of $\triangle AOB = \frac{1}{2} \times \frac{5}{2} \times 5$
$= 6.25$ unit^2

c) $\frac{4x+2}{x+1} = 4 - \frac{2}{x+1}$

$x+1 \overline{\smash{)}4x+2}$ quotient 4
$-(4x+4)$
-2

when $y = 4$
$4 = 4 - \frac{2}{x+1}$
$-\frac{2}{x+1} = 0$
$-2 \neq 0$
$\therefore c \neq 4$